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Latent Markov Models to Test the Strategy Use of 3-Year-Olds in a Rule-Based Feedback-Learning Task

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ABSTRACT
This study is the first to investigate how 3-year-olds learn simple rules from feedback using the Toddler Card Sorting Task (TCST). To account for intra- and inter- individual differences in the learning process, latent Markov models were fitted to the time series of accuracy responses using maximum likelihood techniques (Visser et al., 2002). In a first, exploratory study (N = 110, 3- to 5-years olds) a considerable group of 3-year olds applied a hypothesis testing learning strategy. A second study confirmed these results with a preregistered study (3-years olds, N = 60). Under supportive learning conditions, a majority of 3-year- olds was capable of hypothesis testing. Furthermore, older children and those with bigger working memory capacities were more likely to use hypothesis testing, even though the latter group perseverated more than younger children or those with smaller working memory capacities. 3-year-olds are more advanced feedback-learners than assumed.

KEYWORDS
Feedback-learning; rule learning; 3-year-olds; working memory; latent Markov modeling; hypothesis testing

Introduction

Where many young children display difficulty discovering rules in feedback-learning tasks, this skill rapidly improves with age (Bunge & Zelazo, 2006). Interestingly, the pace of development is not uniform, resulting in some children of the same chronological age applying a more efficient learning strategy than others (Kendler, 1979; Schmittmann et al., 2012). There are multiple learning strategies people employ and the reliance on those underlying strategies changes during development (Ashby et al., 1998; Hanania & Smith, 2010; Huang-Pollock et al., 2011; Kendler, 1979; Schmittmann et al., 2012). The occurrence of a variety of latent learning strategies within age groups and across individuals makes it difficult to reliably reveal children's abilities (Molenaar, 2004). Using latent Markov models, the current study aims to unravel what learning strategies young children employ while learning simple rules (i.e., rules based on one stimulus dimension, such as color) from feedback, and how applying such learning strategies is related to age.

Trial-by-trial accuracy data of a learning episode allows for modeling the learning process by latent Markov models. These models account for inter- and intra-individual differences in this learning process, which are to be expected especially in young children. That is, the latent Markov models allow for the detection of multiple latent strategies, simultaneously represented by one hidden Markov model (Rabiner, 1989).

Hypothesis testing is an effective strategy to learn simple rules from feedback (Schmittmann et al., 2012). In solving a rule-based learning task (that is, categorization tasks that are based on simple rules), hypotheses representing possible rules are generated and tested until the correct rule is found, using a win-stay lose-shift strategy (i.e., if you get positive feedback, stick with the hypothesized rule, if you get negative feedback, switch to a new one; Gholson et al., 1972; Schmittmann et al., 2012). Hypothesis testing is relatively fast and learning is sudden. A typical latent markov model to describe this learning process would include a pre-solution state with chance-level responses and a learned state with (nearly) correct responses. In this formalization, the learning parameter would be the transition probability from the first to the second state.

At which age children start to use a hypothesis testing strategy is still a question of debate. Children as young as 3-years-of-age have the capacity to encode stimuli in rule-based tasks by their separate features - a prerequisite for hypothesis testing (cf. *analytical encoding; Levels of functioning theory*; Kendler, 1979) - theoretically allowing them to use these learning strategies (van Bers et al., 2014; Schwarzer, 2002). However, Minda et al. (2008) report that 4-years olds perform relatively better on tasks that can be solved using more effortful, less efficient types of learning, like learning separate stimulus-response relations. Interestingly, many 4- and 5-year-olds do already seem to rely on a hypothesis testing strategy, even though in a somewhat inefficient fashion (Schmittmann et al., 2012; Schmittmann et al., 2006; Visser et al., 2007). Schmittmann et al. (2012) report on slow learning as an inefficient, discontinuous learning process, that is, a learning process with a sudden change from a pre-solution to a learned state. A latent Markov model could describe the difference between hypothesis testing and slow learning by the value of the transition parameter from a pre-solution state to a learned state. With age, the use of hypothesis testing seems to become more present and applied more efficiently (Schmittmann et al., 2012).

The use of an efficient hypothesis testing strategy is likely linked to the development of executive functioning (Rabi & Minda, 2014; Zelazo, 2006). First, working memory is necessary to store, arrange, select and manipulate hypotheses (Gholson et al., 1972). Substantial working memory involvement is especially important to avoid the interference of latent memory traces of faulty hypotheses with the selection of a new hypothesis, and therefore avoid perseveration (Munakata, 1998). Second, lack of inhibition could hinder the win-stay loose-shift principle in hypothesis testing when irrelevant, not winning, hypotheses cannot be sufficiently inhibited (*Attentional inertia theory*; Kirkham et al., 2003) or previously inhibited hypotheses cannot be re-activated (*Activation deficit account*; Chevalier & Blaye, 2008). Lastly, for effective hypothesis testing, the formulation, application and switching between mental rule sets (e.g. simple rules based on color vs shape) is necessary. Cognitive flexibility is believed to allow for switches between these different rule sets using higher order rules ("If I need to sort on shape, and it's a flower, then it goes here"; CCC-r theory, Zelazo et al., 2003). Limited executive function capacity could thus critically impair hypothesis testing. Interestingly, children rapidly develop their executive abilities between ages 3 and 5 (Carlson, 2005), coinciding with the switch in learning strategy toward hypothesis testing observed in earlier studies in these age groups (Schmittmann et al., 2012). In latent markov models describing the learning process, executive function scores are expected to moderate the transition parameters between states, such that hypothesis testing becomes more efficient with better executive functions.

In sum, around age four children start to use hypothesis testing in rule-based feedback-learning tasks (Schmittmann et al., 2012). Executive functions like working memory, inhibition and cognitive flexibility could potentially be closely related to the ability to apply an efficient hypothesis testing strategy. This study posed the question: How do young children learn simple rules in a rule-based feedback-learning task? Within this general framework, the purpose of this study was twofold: to investigate which learning strategies are employed by pre-schoolers to learn simple rules from feedback and to examine the relation between this strategy-use, age and executive functioning. Both questions were first approached in an exploratory manner in study 1, using a sample of 3- to 5-year-olds. The results obtained were verified in a pre-registered second study (Lichtenberg & Raijmakers 2019) in a new sample exclusively consisting of 3-year-olds.

In the first, exploratory study we develop a parsimonious model by fitting latent Markov models to trial-by-trial accuracy data from children performing a rule-based feedback-learning task. Model-selection techniques provide formal grounds to decide which set of strategies forms the simplest, optimal model to describe the trial-by-trial data (Visser et al., 2002). In the second study, we perform a confirmatory test of this model using a second dataset, collected after pre-registration of the expected model.

Study 1

Method

Participants

The sample contained data from 110 children age three ($n = 42$, $M = 43.0$ months, $SD = 2.2$ months), four ($n = 56$, $M = 54.5$, $SD = 3.7$ months) and five ($n = 12$, $M = 63.7$ months, $SD = 2.8$ months). The total sample consisted of 64 boys and 46 girls. Children were tested in the Babylab from the University of Amsterdam, at schools or in the Nemo Science museum. Informed consent was obtained for all children. Exclusion criteria can be found in the appendix.

Materials

Toddler Card sorting task (TCST)

The main task was a newly developed, computerized feedback-learning task based on the Wisconsin Card

Sort Task (Grant & Berg, 1948), but adapted for young children (Figure 1). The task was programmed in python version 2.7.3 and was administered on a 12-inch HP TouchSmart tm2 touchscreen laptop. Stimuli in this task consisted of cards that differed in shape and color. Target cards matched each reference card on one dimension only (e.g. shape or color). Children had to discover the untold sorting rule (e.g. sort on shape or sort on color) based on the feedback provided. Participants received either of two types of feedback, both containing a verbal component. In the emoticon condition, an additional happy or sad emoticon, corresponding with the correctness of the sort, appeared on screen. In the modeling condition, verbal feedback was combined with the experimenter manually moving the target card toward the correct position after every wrong sort.

This task included three phases: a practice phase, a test phase, and a generalization phase. In the practice phase, children sorted six cards that were exact duplicates of the reference cards to introduce card sorting. In the following phases, the sorting cards matched reference cards on one dimension only. In the test phase, the first sort always triggered negative feedback, which set the sorting rule. It was set to shape if a child started out sorting on color and vice versa, in order to force children to use a win-stay loose-shift principle. This prevented children from reaching the learned state without making any errors because the set rule matched their bias, such as the shape rule in case of a shape bias (Smith & Slone, 2017). Criterion was reached by sorting correctly six cards in a row, with a maximum of 24 cards sorted. Feedback on the accuracy of the sort was provided after every trial. The final phase was a generalization phase with nine trials, in which new stimuli were presented and the child was asked to continue sorting as they did before (Figure 1). No feedback was provided on the generalization trials.

Executive functions

To obtain measures for working memory, inhibition and cognitive flexibility, the *corsi block task* (Bull et al., 2008), *Grass/Snow task* (Carlson & Moses, 2001) and *Dimensional Change Card Sorting Task* (DCCS; Zelazo, 2006) were respectively conducted. Descriptions can be found in the supplementary materials.

Procedure

Children were tested in a single one-on-one session that lasted no longer than 20 minutes. The feedback-

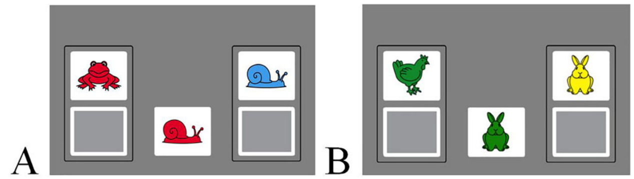

Figure 1. Stimuli and screen set-up of the feedback-learning task in test phase (A) and generalization phase (B). Stimuli differed on two dimensions: shape and color. Children were instructed to sort the target card (e.g. red snail or blue frog) with one of the reference cards (red frog and blue snail). The sorting rule (sort on shape or sort on color) should be discovered using the feedback provided.

learning task and DCCS were administered first, in counterbalanced order. After, the Grass/Snow task and Corsi Block test were administered in this set order. Parents were allowed to be present during testing, but were asked to refrain from interfering.

Statistical approach

Trial-by-trial learning data

Models were created in R statistical software version 3.5.2 (R Core Team, 2017) using the depmixS4 package version 1.3-5 (Visser & Speekenbrink, 2010). Candidate models were selected using AIC and BIC, with lower values indicating better model fit (Akaike, 1974). Likelihood ratio tests were conducted to indicate significant differences between models.

Executive functions

An exploratory factor analysis was applied to the three executive functions. To the learning parameters of the selected latent Markov model, age, and the composite scores of working memory, inhibition, and cognitive flexibility were added as covariates. The model definition is discussed in the Results section (see Figure 2A).

Results

All analyses conducted on this dataset were exploratory in nature.

Descriptives

We analyzed the overall number of errors children made, to see which type of feedback was most beneficial. Children who received modeling feedback made on average significantly fewer errors (after the first, forced error) than those receiving emoticon feedback ($M_{mod} = 0.82$, $SD_{mod} = 1.42$; $M_{emo} = 2.56$, $SD_{emo} =$

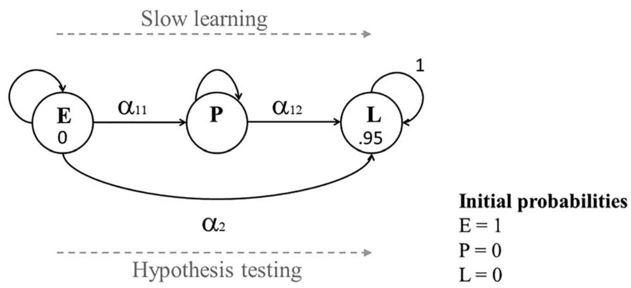

Figure 2. Latent Markov model reflecting possible learning strategies in 3-year-olds. The model consists of three states: an error state (E), a pre-solution state (P) and a learned state (L). Arrows indicate possible transitions between states. Slow learning is characterized by a transition from the error state, through the pre-solution state (α_{11}) to the learned state (α_{12}). Hypothesis testing is indicated by a direct transition from the error state to the learned state (α_2).

3.85; $t(75.60) = -3.22$, $p = .002$) indicating that modeling feedback was more beneficial for them.

Learning strategies

To gain initial insights in 3- to 5-year-olds' strategy use, we fitted four fully unconstrained latent Markov models to the data, with one through four states respectively. Model fit indices indicated that the three-state model fitted the data best (Table A1). We calculated participants' posterior probabilities of visiting states per trial given the three-state model, indicating in which state a participant was likely to be at that trial. It appeared that participants followed either one of two patterns, which could possibly reflect different learning strategies.

Following these observations and the strategies found by Schmittmann et al. (2012), we created a three-state latent Markov model that captured the learning strategies possibly used by 3- to 5-year-olds when learning simple rules from feedback (Figure 2). The model includes two learning routes from the Error state (E, with an accuracy fixed at 0, where by definition the first error is made, and where due to the nature of the task all participants start out) to the Learned state (L, an absorbing state where the children perform with high accuracy, fixed at .95): The direct route with learning parameter α_2, represents hypothesis testing, as long as the wrong hypothesis is followed, the participant is in the E state. Once the correct hypothesis is selected the participant is in the learned state. The indirect route goes from the E state with transition probability α_{11} to the pre-solution state (P where responses are expected to be at chance level, modeled with p_c) and with a transition probability, that is a learning parameter α_{12} from the Pre-solution to the Learned state. The model including both paths fitted the data significantly better than both other, more parsimonious models (both strategies vs hypothesis only: $\Delta x^2(1) = 4.432$, $p = .035$; both strategies vs slow only: $\Delta x^2(1) = 83.124$, $p < .000$). This model will henceforth be referred to as Base Model (BM).

The presence of two distinct strategies for learning indicates that some 3- to 5-year-olds learn simple rules from feedback using hypothesis testing, while others use a slow learning strategy. Children following a slow learning strategy that do not reach criterion within 24 trials are referred to as non-learners.

Response probability

Based on theoretical constraints, response probabilities in the error and learned states were fixed at 0 and .95 respectively (Schmittmann et al., 2012). We hypothesized that response probabilities in the pre-solution state would be at chance level (.50), reflecting a trial-and-error type of process. Freely estimating the response probability in the pre-solution state did not significantly improve model fit compared to a model where this parameter was fixed at .50, that is chance level (BM-fix; $\Delta x^2(1) = 3.085$, $p = .079$). The final model is displayed in Figure 3.

Individual strategies

To estimate what proportion of children used a certain strategy, we inspected the posterior probabilities per trial per child given the BM-fix model (Table A2). A clear developmental trend seems present, where children become more efficient learners when they grow older. Whereas an already surprisingly large proportion of 3-year-olds used the hypothesis testing strategy (72%), almost all 4-year-olds (97%) and all 5-year-olds did so. The type of feedback children received seemed to influence the efficiency with which they learned, as indicated by a significant difference in strategy use between feedback conditions ($t(66.7) = -3.236$, $p = .002$). When receiving modeling feedback, all 3-year-olds reached criterion within 24 trials. In 4-year-olds, there are no slow learners left, indicating modeling feedback might have helped them to reach their full potential (for an overview of mixing proportions per feedback condition, see Appendix).

Generalization

Sorting behavior in the generalization phase of the task was investigated to see if children following different learning processes in the test phase also adhered to distinct response strategies during this phase. Eighty percent of hypothesis testing children continued their test-phase categorization (e.g., a color rule) in the generalization phase, strengthening the evidence for the presence of rule-based representations of the learned categorization. (See appendix for a more detailed description of the generalization analysis).

Relating executive functioning to learning strategies

Startegies and the executive construct

The EF measures were combined into a single measure based on a factor analysis (see Appendix A for details). Both the composite measure and standardized age in months were added to the BM-fix model as covariates to determine if EF was associated with the learning strategies applied in addition to age. The two covariates (executive functions, EF, and age, A; A&EF) were either added to both learning parameters (BM-fix-A&EF-both), only to the hypothesis testing parameter (BM-fix-A&EF-hyp), or only to the slow learning parameter (BM-fix-A&EF-slow). In the following analysis we tested whether WM and FL were explaining variance in the data in addition to the contribution of age, which is a conservative test. The BM-fix-A&EF-hyp model fitted the data best (see Table A4 for fit indices), meaning that Age and EF significantly influence the hypothesis testing parameter. This model fitted significantly better than the BM-fix model ($\Delta x^2(3) = 11.006$, $p = .010$), but also better than the BM-fix-A&EF-slow model at $\Delta x^2(0) = 4.208$, $p < .001$. There was no significant difference between the BM-fix-A&EF-both and BM-fix-A&EF-hyp model ($\Delta x^2(6) = 5.823$ $p = .44$), altogether indicating that there was no association between slow learning and the covariates.

The association between the covariates and strategy use (i.e. the hypothesis testing parameter) was driven by the executive functions (BM-fix vs BM-fix-EF-hyp, $\Delta x^2(1) = 8.374$, $p = 0.004$), but not significantly by age (BM-fix vs BM-fix-A-hyp, $\Delta x^2(1) = 3.051$, $p = .08$). The BM-fix-A&EF-hyp model fitted the data significantly better than the BM-fix-A-hyp model ($\Delta x^2(-2) = 7.955$, $p = .019$), indicating that executive functions did explain variance in strategy use in addition to age.

Coefficients of the BM-fix-EF-hyp model indicated that the executive construct was positively associated with hypothesis testing (α_2: $\beta = .709$), and negatively with the slow learning strategy (α_{11}: $\beta = -0.233$). Together these results indicate that children with more developmentally advanced executive functions are more likely to use a hypothesis testing strategy.

Conclusion

Study 1 aimed to explore how 3- to 5-year-olds learn simple (i.e. one-dimensional) rules from feedback. We fitted latent Markov models to the trial-by-trial accuracy data of 3- to 5-year-olds performing a feedback-learning task. We found indications that 3- to 5-year-olds used different learning strategies when learning simple rules from feedback: slow/non learning and hypothesis testing. Results suggest that contrary to common belief (Ashby et al., 1998; Gholson et al., 1972; Minda et al., 2008; Schmittmann et al., 2012), many 3-year-olds might already be capable of learning from feedback using hypothesis testing. Moreover, children with further developed executive functions were more likely to use a hypothesis testing strategy. That is, executive functions have a specific predictive value in strategy use, in addition to age.

Limitations and future directions

Study 1 was exploratory in nature. Replication in a new sample is needed before strong conclusions can be drawn. Based on the findings that hypothesis testing already seemed more common in 3-year-olds than previously believed (Ashby et al., 1998; Gholson et al., 1972; Minda et al., 2008; Schmittmann et al., 2012), we decided to focus our next study solely on this still understudied age group.

Based on observations in study 1, we made some adjustments. First, we will only include modeling feedback, as young children seemed to profit most from this type of feedback. Second, in the current two-option set-up of the task simple deduction could technically also be used to discover the sorting rule (e.g. if it is not A, it must be B). To infer hypothesis testing with more certainty, a third dimension needed to be added to the TCST, making it a three dimensional task (3 D-TCST). We opted for a number dimension (as taken from the Wisconsin Card Sorting Task), since young children can differentiate between the quantities one, two and three by subitizing (Bruce & Threlfall, 2004; Grant & Berg, 1948), allowing for such a dimension to be implemented in tasks for the very youngest.

Study 2

Study 2 first aimed to confirm the existence of multiple learning strategies (hypothesis testing and slow learning) in 3-year olds. Second, the study aimed to confirm the association between age and the executive functions in applying a hypothesis testing strategy.

We expected the hypothesis testers to be the older children in the sample, and the ones with more developmentally advanced executive functions (Ashby et al., 1998; Gholson et al., 1972; Zeithamova & Maddox, 2007). To this end, we preregistered the analyses that consist of first fitting the Base Model to the trial-by-trial data and subsequently adding the executive functions and age as covariates to the learning parameters.

Method

Participants

A total of 62 3-year-olds took part in this study, ages ranging from 34.7 months to 48.6 (M_{months} = 41.77, SD_{months} = 3.95). There were 35 boys and 27 girls. Children were recruited via the University's Babylab database, social media and personal networks. They were tested in the Babylab (n = 31), at home (n = 10) or at a daycare facility (n = 21). During the session, children earned six stickers they were allowed to bring home and all children, except those tested at the daycare facility (upon special request of the facility), were given a small gift, such as bubble blow, after participation as a reward. Informed consent was obtained for all the children by a parent or primary caregiver. Exclusion criteria can be found in the appendix.

Materials

Three Dimensional - Toddler card sorting task (3 D-TCST)

The main task was an adapted version of the original TCST used in study 1 with two exceptions: stimuli on task and reference cards differed on three dimensions - shape, color and number - to infer hypothesis testing with more certainty (see figure 4 for stimuli and task set-up), and only modeling feedback was provided as children seemed to profit from this feedback most. The generalization phase that directly follows the test phase (as in study 1) diagnoses the rule that children apply at the end of the learning process (cf., Molenaar et al., 2014). The outcome can be seen as a validation of the learning task, such that children who generalize the learned categorization by the application of a simple rule, learned a simple rule in the first place (instead of, e.g., separate stimulus-response relations).

Executive functions

To measure children's executive functioning, children completed three independent tasks: *Spin the pots* for working memory (Huges & Ensor, 2005), the *Day-Night-task* for inhibition (Carlson & Moses, 2001), and the *Dimensional Change Card Sorting Task* (DCCS) for cognitive flexibility (Zelazo, 2006). Full task descriptions can be found in Appendix B.

Procedure

All children were tested in a single one-on-one session, generally lasting between 30 and 45 minutes. Parents were allowed to be present during testing, but explicitly asked to refrain from interfering. The tasks were administered in a fixed order: feedback-learning, spin the pots, Day/Night and DCCS. This order prevented possible crossover effects between the two rule-based learning tasks and created optimal variation in the test session, alternating between tasks of different nature and with different media, hence keeping the children engaged and concentrated. In between tasks, breaks of variable duration, but no longer than 10 minutes, were inserted when needed.

Statistical approach

The analysis plan was pre-registered on the Open Science Framework (Lichtenberg & Raijmakers, 2019), with the exception of the generalization analyses. The base model (BM) is used as the standard model to fit to the trial-by-trial learning data. The BM is compared to an only hypothesis testing model, an only slow-learning model and a BM-fix (with a response probability in the presolution state of .33). Analyses were executed as planned, with the exception of the Exploratory Factor Analysis for the executive function tasks. Due to unforeseen responses and missing data of a substantial number of toddlers on the inhibition task, we decided to drop this task from our analyses (inflated number of missing data, which was likely not missing at random). The remaining two executive function tasks did not share enough variance to perform an Exploratory Factor Analysis, therefore remaining analyses were run for working memory and cognitive flexibility separately.

Results

Learning strategy

Feedback-learning strategies

To confirm that 3-year-olds use multiple learning strategies, we fitted three versions of the latent Markov model representing the expected underlying structures of 3-year-olds' feedback-learning strategies,

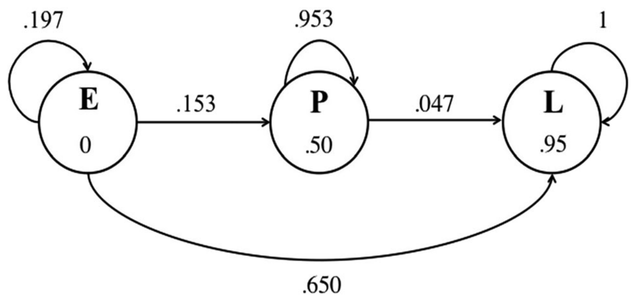

Figure 3. The final, base model (BM-fix) with likelihood estimates of the parameters. Response probabilities of answering correctly are displayed within the states. n = 110.

to the trial-by-trial accuracy data. As expected, a three-state model with both a fast (α_2) and slow learning parameter (α_{12}), the BM, fitted the data best (Figure 5A, Table B2). This model fitted the data significantly better than both more parsimonious models (both vs hypothesis only: $\Delta x^2(1) = 7.200$ $p = .007$; both vs slow only: $\Delta x^2(1) = 41.974$, $p < .001$), confirming our hypothesis that some 3-year-olds learn simple rules from feedback using a hypothesis testing strategy, while others use a slow learning strategy.

Response probability

In contrast to study 1, results suggest that the response probability in the pre-solution state was not at chance level (chance = .33; $\Delta x^2(1) = 8.393$, $p = .004$). That is, slow learners performed well above chance, with a probability of .44 to sort a card correctly . The final model, optimized to the data, is displayed in Figure 5B.

Individual strategies

To reveal how many 3-year-olds used which strategy, we inspected for all children the posterior probabilities of following each learning strategy. With the 3 D-TCST, 78.3% (n = 47) of the 3-year-olds used the hypothesis testing strategy ($n_{slow\ learning} = 3$, 5.0%; $n_{non\text{-}learning} = 10$, 16.7%). This shows that, when provided with the learning conditions that seemed most beneficial to children in study 1, hypothesis testing is in reach for many 3-year-olds.

Generalization

As in study 1, the majority of hypothesis testing children continued their learning strategy into the generalization phase (Table B2 shows model selection results of the mixture distribution analysis of the generalization data). Sixtyfour percent continued applying the learned rule with high accuracy ($p = .99$) and 36% with lower accuracy ($p = .72$; different from chance level, i.e. $p = .33$).

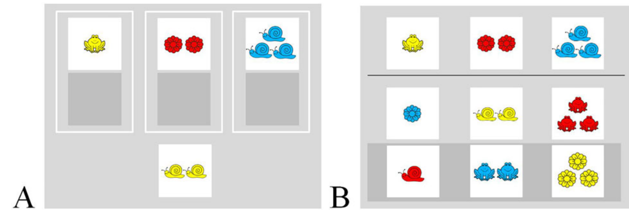

Figure 4. Stimuli and screen set-up (A) and stimuli (B) of the feedback-learning task used in study 2. Stimuli differed on three dimensions: shape, color and number. Children were instructed to sort one of the target cards (B: middle row) with one of the reference cards (B: top row). The sorting rule could be discovered using the feedback provided after every trial. In the generalization phase, the target cards were replaced by stimuli from the bottom row (B).

Relating executive functioning to learning strategy

Executive functions and strategies. To see whether there was an association between learning and age and executive functions, age (A), working memory (WM) and cognitive flexibility (FL) were added as covariates to the learning parameters of the selected latent Markov model. In the following analysis we tested whether WM and FL were explaining variance in the data in addition to the contribution of age, which is a conservative test. In DepmixS4, the response parameter in the pre-solution state could not be freely estimated simultaneously with adding covariates to the model. The value obtained when optimizing the BM model to the data was used as a fixed value in further models (BM-fix). Again, the covariates were either added to both learning parameters (BM-fix-WMA-both/BM-fix-FLA-both), only the hypothesis testing parameter (BM-fix-WMA-hyp/BM-fix-FLA-hyp), or only the slow learning parameter (BM-fix-WMA-slow/BM-fix-FLA-slow).

Analyses show that, cognitive flexibility was not significantly associated with either learning parameter (BM-fix vs BM-fix-FLA-hyp, $\Delta x^2(3) = 5.962$, $p = .114$; BM-fix vs B<-fix-FLA-slow, $\Delta x^2(3) = 0.799$, $p = .850$). In contrast, working memory and age were significantly associated with the use of hypothesis testing (BM-fix vs BM-fix-WMA-hyp, $\Delta x^2(3) = 9.323$, $p = .025$). Adding the covariates to the slow learning parameter did not significantly improve model fit (BM-fix vs BM-fix-WMA-both, $\Delta x^2(9) = 17.088$, $p = .047$), indicating that the efficiency of slow learning was not associated with the covariates. See Table B3 for model fit and comparison indices.

Both covariates, A and WM, established an association with the hypothesis testing parameter by themselves, indicating that the effect of neither of them was negligible (BM-fix vs BM-fix-A-hyp, $\Delta x^2(1) = 5.096$, $p = .024$, BM-fix vs BM-fix-WM-hyp, $\Delta x^2(1) =$

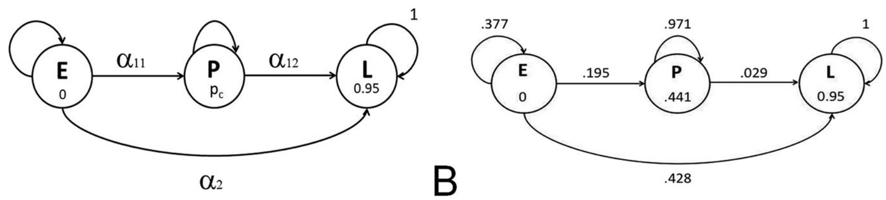

Figure 5. A: The Base Model with four free parameters, α_{11}, α_{12}, α_2, and p_c. B: The final model (BM) optimized to the data. The model contains three states, an error state (E) a presolution state (P) and a learned state (L). Arrows indicate possible transitions with transition probabilities. Response probabilities of answering correctly are displayed within the states.

6.744, $p = .009$). As expected, the combined model did not fit the data better than the separate models (BM-fix-WMA-hyp vs BM-fix-A-hyp, $\Delta x^2(2) = 4.227$, $p = .121$; BM-fix-WMA-hyp vs BM-fix-WM-hyp, $\Delta x^2(2) = 2.575$, $p = .275$). The correlation between age and working memory might partially account for this overlap ($r(44) = .35$, $p = .018$). The AIC and BIC prefer the BM-fix-WM-hyp model, but the significance of the difference cannot be tested. See Table B4 for model fit and comparison indices.

The coefficients of the BM-fix-A-hyp model indicated that age was positively associated with the hypothesis testing parameter (α_2: $\beta = .49$; see Table B5), suggesting that older children are more likely to use this strategy. Logically following this observation, age was negatively associated with the slow learning component (α_{11}: $\beta = -0.29$), as was the case for working memory in the BM-fix-WM-hyp model (α_{11}: $\beta = -1.44$; see Table B6). This indicates that both older children and those with bigger working memory capacities were less likely to be slow learners. Interestingly, working memory was also negatively associated with the hypothesis testing parameter (α_2: $\beta = -0.17$), reducing the transitional probability on individual trials from the error state directly to the learned state for children with higher working memory scores. This negative association indicates more perseveration for hypothesis testers with higher WM scores.

Conclusion and discussion

The aim of this study was twofold: to verify the existence of multiple learning strategies in 3-year-olds and to see how these strategies might be related to the development of the executive functions of these children. Following our preregistration on the Open Science Framework (Lichtenberg & Raijmakers, 2019), we fitted latent Markov models on trial-by-trial accuracy data to test whether 3-year-olds used different learning strategies to learn from feedback, that is, slow learning and hypothesis testing. Results showed that hypothesis testing was already in reach for many 3-year-olds. Working memory and age were both associated with strategy use, such that more developmentally advanced and older children were more likely to use a hypothesis testing strategy. But hypothesis testers with larger working memory perseverate their initial, incorrect responses longer.

Strategy use

Typical results indicate that hypothesis testing in feedback-learning paradigms is only used by children older than three years of age (Ashby et al., 1998; Gholson et al., 1972; Minda et al., 2008; Schmittmann et al., 2012). Design differences between the current study and previous work can possibly account for the differences in learning strategies observed. We used a simple task structure (3 D-TCST) and modeling feedback in order to ensure that children's executive functions were not unnecessarily taxed, although they are always involved in rule learning.

In contrast, previous studies have often used more complex tasks, such as rule-based categorization tasks that pose a bigger constraint on working memory due to the absence of reference stimuli (Minda et al., 2008), rule search and application tasks where rules should first be selected based on an arbitrary stimulus and then applied to target stimuli (Van Duijvenvoorde et al., 2008) or discrimination learning tasks that are based on dimension comparison instead of dimension matching, such as the discrimination-learning task (Kendler, 1979; Schmittmann et al., 2012).

Besides task complexity, the type of feedback provided varies drastically over studies. Where some studies only provided nonverbal feedback (Minda et al., 2008), others only provided feedback on certain trials (Gholson et al., 1972). Most importantly, some tasks entail false positive feedback, creating a situation in which children possibly believe they are right for the wrong reason (i.e. based on the irrelevant dimension; Kendler, 1979; Schmittmann et al., 2012). False positive feedback majorly complicates the hypothesis testing process by rendering the win-stay principle unreliable. Altogether, differences in task complexity and feedback information content could have previously hindered hypothesis testing of 3-year-olds.

Executive functions

Age and working memory, but not cognitive flexibility, were related to strategy use, such that older children and children with more developmentally advanced working memory were more likely to use a hypothesis testing strategy. Interestingly, the associations for age and working memory were only partly interchangeable. Where older children were just more likely to use a hypothesis testing strategy, children with bigger working memory capacities on average perseverated on more trials in the error state before transitioning directly to the learned state, indicating less efficient use of hypothesis testing. Due to the nature of the task (starting with negative feedback), especially children with better working memory could have more difficulty to abandon their preferred, falsified rule, resulting in perseveration (Munakata, 1998; cf. Schmittmann et al., 2012).

The current study focused mainly on the hypothesis testing strategy and could not determine in more detail what strategy is exactly applied by slow/non-learners. Theories that could explain slow and non-learning vary from incremental or associative learning (Ashby et al., 1998; Kendler, 1979), to the inefficient use of (a mix of) up-to-date still undetermined strategies (Inkster et al., 2014; Schmittmann et al., 2006).

Peter Molenaar's legacy and future directions

The detailed information about 3-years olds hypothesis testing could only be revealed by the application of statistical models that account for inter-individual differences (due to variation in strategies and order of tested hypotheses) and intra-individual differences (accuracies changed suddenly during learning). Peter Molenaar (2004) extensively argued that inter-individual variation is not generalizable to intra-individual variation. The line of work continued in this article, the application of hidden Markov models to psychological data (Visser et al., 2002), does not focus on modeling individual time series but on latent mixtures of time series. However, these models do assume that variation between individuals (e.g., transition probabilities between Markov states) are generalizable to within-individual processes (e.g., learning speed). The advantage of modeling a limited number of shorter time series is evident in toddler research. Especially in the domain of cognitive development with young children where data collection is effortful for participants and researchers, latent Markov modeling provides an excellent technique to reliably test detailed hypotheses about learning processes accounting for important inter and intra-individual differences. Techniques have been improving considerably since the first applications to psychological data (Visser & Speekenbrink, 2022).

An important future direction of this research is to design a longitudinal study about the development of learning processes. As discussed in the Manifesto (Molenaar, 2004), the variance within an individual, that is, variation of learning strategies over time during development, is not necessarily similar to the variance between individuals, that is learning strategies in a cross-sectional sample of children with different ages. Remarkably enough, there are not many datasets of high density, longitudinal data of preschoolers (cf., Stifter & Rovine, 2015). These data could give us some insight to contribute to the challenge formulated by Coenen et al. (2019): "What is the developmental trajectory of inquiry abilities?".

Conclusion

In sum, using latent Markov models we showed that 3-year-olds used two distinct learning strategies when learning simple (i.e. one dimensional) rules from feedback in the 3D-TCST: slow leaning and hypothesis testing. Together, these results call for a revalidation of the feedback-learning capacities of 3-year olds. When provided with supportive learning conditions like a simple task structure and informative feedback, a majority already used a hypothesis testing strategy, making them more advanced feedback-learners than previously believed.

Open practices statement

This confirmatory study was preregistered at the Open Science Framework (Lichtenberg & Raijmakers, June 19). Feedback-learning in 3-year-olds; osf.io/yaexv). Together with the preregistration, the data and latent Markov model scripts will be made publicly available on this Open Science Framework page.

Article information

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Ethical principles: The authors affirm having followed professional ethical guidelines in preparing this work. These guidelines include obtaining informed consent from human participants, maintaining ethical treatment and respect for the rights of human or animal participants, and ensuring the privacy of participants and their data, such as ensuring that individual participants cannot be identified in reported results or from publicly available original or archival data.

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Appendix A– Tasks and tables exploratory study

Study 1

Exclusion criteria

Exclusion criteria

All children from whom feedback-learning data was available were included in the feedback-learning strategy and generalization analyses (n = 110). Executive function data was available for 93 children, who were all included in the Exploratory Factor Analysis. Thirteen participants were excluded from the covariates analysis for failing the pre-switch phase of the Dimensional Change Card Sort task (DCCS), resulting in a final sample of 80 participants.

Table A1. Model fit indices.

Model	LogL	AIC	BIC	*df*
BM	**−443.21**	**894.43**	**914.58**	**4**
Hyp only	−445.43	896.86	911.98	3
Slow only	−484.78	975.55	990.67	3
BM-fix	**−444.76**	**895.51**	**910.63**	**3**

Note. Model fit indices of latent Markov models of trial-by-trial data. BM: base model containing learning parameters reflecting both strategies, Hyp only: only hypothesis testing, Slow only: only slow learning, and BM-fix: base model with response probability in the pre-solution state fixed at chance (.50). n = 110.The fixed base model (a 3-state model with both a fast and slow learning parameter and the response parameter of the pre-solution state fixed at chance) fitted the data best.

Table A2. Proportions per strategy per age group given the BM-fix model.

	3-year-olds		4-year-olds		5-year-olds	
	N	Proportion	N	Proportion	N	Proportion
Hypothesis testers	30	.715	55	.965	11	1.000
Slow learners	4	.095	0	0	0	0
Non-learners	8	.190	2	.035	0	0

Note. n = 110.

Test phase data analysis

Generalization data analysis (confirmatory)

Generalization

Generalization of the learned rule to new stimuli shows the way in which the learned rule is represented. To the sum scores of the generalization phase (with respect to continuing the correct rule of the test phase) of 110 children, we fitted four binomial mixture models, with one to four components respectively. The mixture model with three binomial distributions provided the most optimal fit for our data and also provided a good absolute fit ($x^2(36) = 42$, $p = .227$; see Table A.2). Children either sorted all cards correctly, that is, in line with the learned rule, (probability correct, p = .98), performed at chance level (p = .57), or sorted all cards incorrectly (p = .07).

As predicted, most hypothesis testers continued to apply the learned rule in the generalization phase (n = 77, 80%). A smaller portion seemed to have lost their rule and performed at chance level (n = 10, 11%). The remaining hypothesis testers consistently sorted generalization cards incorrectly (n = 9, 9%). The majority of slow and non-learners expectedly performed at chance level (n = 7, 50%) or worse (n = 4, 29%). Only a small group sorted all generalization cards correctly (n = 3, 21%). These results are in line with the idea that the strategies found in the learning task reflect learning a simple rule, since the majority of children extrapolated their strategy to the generalization phase.

Table A3. Model fit indices of the generalization models.

Model	LogL	AIC	BIC	df
1-state	−258.8141	519.6281	522.3286	1
2-state	−151.8008	309.6015	317.7030	3
3-state	**−145.8982**	**301.7964**	**315.2988**	**5**
4-state	−144.7242	303.4485	322.3518	7

Note. Model fit indices mixture distribution of binomials models generalization data. n = 110.

Executive function analysis

Working memory

Corsi block task was administered to measure childrens'-working memory capacities (Bull et al., 2008). Children were shown a physical array of identical blue blocks. The experimenter tapped predetermined patterns on the blocks. The child was then asked to repeat the pattern by tapping the same order. When patterns were repeated correctly, pattern length increased over trials. Longer correctly repeated patterns were taken as indications of bigger working memory spans.

Inhibition

The *Grass/Snow-task* was administered to measure a child's inhibition capacities (Carlson & Moses, 2001). Children were shown two colored rectangles on a tablet and were asked if they knew which one represented 'Grass' (i.e. the green color slab) and which one represented 'Snow'(i.e. The white color slab). Subsequently, they had to inhibit their automatic response by pointing to the white slab when the experimenter said the word 'Grass' and to the green slab when she said 'Snow'. The task advanced with a practice phase of seven trials during which feedback was provided. A child passed the practice phase if each of the stimuli was pointed to correctly at least once on two consecutive trials. During the test phase, another 16 trials were administered without feedback. Higher scores indicated better inhibition capacities.

Cognitive flexibility

The Dimensional Change Card Sorting task (DCCS) measures cognitive flexibility (Zelazo, 2006). In this computerized task, children sorted six 2-dimensional cards given a given sorting rule (e.g. sort on color or sort on shape; pre-switch phase). Subsequently, they were asked to switch to a new sorting rule (i.e. sorting on the other dimension) for the remaining six cards (post-switch phase). The sorting rule was repeated before every trial, but feedback was only provided in the pre-switch phase. Typically, 3- and 4-year-olds pass the pre-switch phase, but fail the post-switch phase by perseverating on the previous sorting rule (van Bers et al., 2014). Switching to the new sorting rule indicated higher cognitive flexibility. Following standard DCCS analysis procedure, post switch scores from this task were converted to a binomial variable (e.g. van Bers et al., 2014; Schmittmann et al., 2012): Children passed the task when sorting at least five out of six post-switch cards correctly, otherwise a *failed* was obtained.

Exploratory Factor analysis of the executive construct

Since there is discussion in the literature regarding the separability of the executive functions in young children (Carlson, 2005; Wiebe et al., 2011; Shing et al., 2010), we conducted an exploratory factor analysis on standardized and centered executive function scores. An oblique oblimin rotation was used to allow possible factors to correlate (Osborne, 2015). The results indicated that the executive functions in our sample all loaded on the same construct: Parallel analysis suggested only one factor had an eigenvalue higher than .7 (Jolliffe, 1972); In the two-factor model, one factor explained 94% of the total variance with factor loadings between .59 and .30, thus the second factor added minimal value. Final factor loadings are displayed in the appendix. A composite measure was created out of the three executive function scores by multiplying the standardized and centered scores with the corresponding factor loadings as weights, and adding the components. For all further analyses, the executive functions were treated as one construct by using this composite score.

Table A4. Factor loadings Exploratory Factor Analysis.

	1 factor	2 factors		3 factors		
Factor nr	1	1	2	1	2	3
WM	**.65**	.59	.06	.58	.06	0
Inhibition	**.44**	.50	−.13	.51	−.11	0
Flexibility	**.29**	.30	.16	.28	.15	0

Note. Factor loadings for one, two and three factor solutions. n = 93.

Covariate analysis

Table A5. Model fit indices and likelihood ratio difference scores compared to BM-fix.

Model	AIC	BIC	*df*	LogL	Δ*df*	ΔLogL	*p*
BM-fix	577.69	591.69	3	−285.84			
BM-fix-A&EF-both	578.82	634.84	12	−277.41	9	16.834	.051
BM-fix-A&EF-hyp	572.66	600.67	6	−280.33	3	11.006	.012*
BM-fix-A-hyp	576.64	595.31	4	−284.32	1	3.051	.081
BM-fix-EF-hyp	**571.32**	**589.99**	**4**	**−281.66**	**1**	**8.374**	**.004***
BM-fix-A&EF-slow	576.93	604.94	6	−282.47	3	6.798	.079

Note. Model fit indices and likelihood ratio difference scores from various models where the executive function (EF) scores and age (A) were added as covariates to both or either learning parameter(s) (both/hyp/slow). n = 80. * $p < .05$.

Table A6. Model fit indices and likelihood ratio difference scores compared to BM-fix-EFA-hyp.

Model	AIC	BIC	df	LogL	Δdf	ΔLogL	p
BM-fix-EFA-hyp	572.66	600.67	6	−280.33			
BM-fix-A-hyp	576.64	595.31	4	−284.32	−2	7.955	.019*
BM-fix-EF-hyp	**571.32**	**589.99**	**4**	**−281.66**	**−2**	**2.632**	**.268**

Note. n = 80. * p < .05.

Table A7. Model coefficients covariate analysis BM-fix-EF-hyp.

	Error	Presolution	Learned
Intercept	0	−.889	1.115
Executive construct	0	−.233	.709
Zero values cov	.224	.092	.684

Note. b-coefficients for the different covariates added to the transition parameters of the error state. Remaining in the error state (Perseveration), transitioning to the presolution state (slow learning) or the learned state (hypothesis testing). Coefficients are displayed for intercept, covariate executive construct and finally the probabilities at zero values of the covariates. n = 80.

Appendix B– Tasks and tables confirmatory study

Study 2

Exclusion criteria

Exclusion criteria

Two participants were excluded from the feedback-learning strategy (and subsequent) analysis due to technical malfunctions in the 3 D-TCST, resulting in a total sample of 60 participants. A further 14 participants were excluded from the covariate analysis (n = 46), due to missing executive function data.

Test phase data analysis

Table B1. Model fit indices.

Model	LogL	AIC	BIC	*df*
BM	**−374.93**	**757.85**	**776.27**	**4**
Hyp only	−378.53	763.05	776.87	3
Slow only	−395.91	797.83	811.64	3
BM-fix	−379.12	764.24	778.06	3

Note. Model fit indices of latent Markov models of trial-by-trial data. BM: base model containing learning parameters reflecting both strategies, Hyp only: only hypothesis testing, Slow only: only slow learning, and BM-fix: base model with response probability in the presolution state fixed at chance, i.e. .33. n = 60. The base model (a 3-state model with both a fast and slow learning parameter) fitted the data best.

Generalization data analysis

Generalization (exploratory)

Generalization of the learned rule to new stimuli shows the way in which the learned rule is represented. To the sum scores of the generalization phase (with respect to continuing the correct rule of the test phase) of 56 children, we fitted four binomial mixture models, with one to four components respectively. The mixture model with three binomial distributions provided the most optimal fit for our data and also provided a good absolute fit ($x^2(36) = 40$, $p = .297$; see Table B.2). Children either sorted all cards correctly, that is, in line with the learned rule, (probability correct, p = .99), performed at chance level (p = .33), or somewhere in between at an average of 72% correct (p = .72).

As predicted, most hypothesis testers continued to apply the learned rule in the generalization phase (n = 29, 64%). A smaller portion had more difficulty in the generalization phase, but still performed well above chance level (n = 16, 36%). The majority of slow and non-learners expectedly performed at chance level (n = 7, 64%). Only a small group performed above chance level at 72% correct (n = 4, 36%). These results are in line with the idea that the strategies found in the learning task reflect learning a simple rule, since the majority of children extrapolated their strategy to the generalization phase.

Table B2. Model fit indices of the generalization models.

Model	LogL	AIC	BIC	*df*
1-state	−184.78	371.57	373.61	1
2-state	−110.24	226.48	232.60	3
3-state-a	−101.10	212.20	222.42	5
3-state-b	**−102.91**	**211.83**	**217.96**	**3**
4-state	−98.77	211.54	225.84	7

Note. Model fit indices of mixture distribution of binomials models of the generalization data. n = 56. 3-state-a: all parameters freely estimated, 3-state-b: one response parameter fixed at .33 correct (chance level).

Executive function analysis

Working memory

To measure working memory, children completed the *Spin the pots task* (Huges & Ensor, 2005). In this task, eight pots of distinct shapes, sizes and colors were displayed on a spinning tray. Together with the experimenter, the child hid six stickers in the pots, leaving two of them empty. The pots were covered up with an opaque scarf and spun around. After the scarf was lifted, the child was allowed to open one of the pots to see if there was a sticker inside. The pot was closed again after every trial. This process was repeated until all stickers had been retrieved or a maximum of 16 trials had passed. Lower scores indicated better working memory capacities. During analysis, performance on this task was reverse scored so that higher scores also indicated better working memory, improving interpretability.

Inhibition

The *Day/Night-task* was administered to measure a child's inhibition capacities (Carlson & Moses, 2001). Children were shown two images and asked if they knew which one represented 'Day' (i.e. picture of the sun) and which one represented 'Night' (i.e. picture of the moon). Subsequently, they had to inhibit their automatic response by pointing to the moon when the experimenter said the word 'Day' and to the sun when she said 'Night'. The task advanced with a practice phase of seven trials during which feedback was provided. A child passed the practice phase if each of the stimuli was pointed to correctly at least once on two consecutive trials. During the test phase, another 16 trials were administered without feedback. Higher scores indicated better inhibition capacities.

Cognitive flexibility

The Dimensional Change Card Sorting task (DCCS) measures cognitive flexibility (Zelazo, 2006). In this computerized task, children sorted six 2-dimensional cards given a given sorting rule (e.g. sort on color or sort on shape; pre-switch phase). Subsequently, they were asked to switch to a new sorting rule (i.e. sorting on the other dimension) for the remaining six cards (post-switch phase). The sorting rule was repeated before every trial, but feedback was only provided in the pre-switch phase. Typically, 3- and 4-year-olds pass the pre-switch phase, but fail the post-switch phase by perseverating on the previous sorting rule (van Bers et al., 2014). Switching to the new sorting rule indicated higher cognitive flexibility. Following standard DCCS analysis procedure, post switch scores from this task were converted to a binomial variable (e.g. van Bers et al., 2014; Schmittmann et al., 2012): Children passed the task when sorting at least five out of six post-switch cards correctly, otherwise a *failed* was obtained.

Covariate analysis

Table B3. Model fit indices compared to BM-fix.

Model	AIC	BIC	*df*	LogL	Δ*df*	ΔLogL	*p*
BM-fix	511.42	524.27	3	−252.71			
BM-fix-WMA-both	512.33	563.72	12	−244.17	9	17.088	.047*
BM-fix-WMA-hyp	508.10	533.79	6	−248.05	3	9.323	.025*
BM-fix-A-hyp	**508.33**	**525.46**	**4**	**−250.16**	**1**	**5.096**	**.024***
BM-fix-WM-hyp	**506.68**	**523.80**	**4**	**−249.34**	**1**	**6.744**	**.009***
BM-fix-WMA-slow	513.67	539.36	6	−250.83	3	3.753	.289
BM-fix-FLA-both	523.29	574.67	12	−249.64	9	6.134	.726
BM-fix-FLA-hyp	511.46	537.15	6	−249.73	3	5.962	.114
BM-fix-FLA-slow	516.62	542.32	6	−252.31	3	.799	.850

Note. Model fit indices and likelihood ratio difference scores from various models where the executive function (WM/FL) scores and age (A) were added as covariates to both or either learning parameter(s) (both/hyp/-slow). * $p < .05$. n = 46. All models are compared to the BM-fix. The models where age and working memory were separately fitted on the learning parameter of the hypothesis-testing strategy fitted the data best.

Table B4. Model fit indices and likelihood ratio difference scores compared to BM-fix-WMA-hyp.

Model	AIC	BIC	*df*	LogL	Δ *df*	Δ LogL	*P*
BM-fix-WMA-hyp	508.10	533.79	6	−248.05			
BM-fix-A-hyp	**508.33**	**525.46**	**4**	**−250.16**	**2**	**4.227**	**.121**
BM-fix-WM-hyp	**506.68**	**523.80**	**4**	**−249.34**	**2**	**2.575**	**.275**

Note. n = 46. * $p < .05$.

Table B5. Model coefficients covariate analysis BM-fix-A-hyp.

	Error state	Presolution state	Learned state
Intercept	0	−1.254	−.005
Age	0	−.289	.493
Zero values cov	.439	.125	.436

Note. β-coefficients for the transition model for the error state (E) of the *BM-fix-A-hyp*. Coefficients are displayed for intercept, the covariate age and finally the probabilities at zero values of the covariates. n = 46.

Table B6. Model coefficients covariate analysis BM-fix-WM-hyp.

	Error state	Presolution state	Learned state
Intercept	0	−1.273	.090
WM_rev	0	−1.437	−.171
Zero values cov	.421	.118	.461

Note. β-coefficients for the transition model for the error state (E) of the *BM-fix-WM-hyp*. Coefficients are displayed for the intercept, the covariate working memory (reverse scored) and finally the probabilities at zero values of the covariates. n = 46.