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Nodewise Parameter Aggregation for Psychometric Networks

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ABSTRACT
Psychometric networks can be estimated using nodewise regression to estimate edge weights when the joint distribution is analytically difficult to derive or the estimation is too computationally intensive. The nodewise approach runs generalized linear models with each node as the outcome. Two regression coefficients are obtained for each link, which need to be aggregated to obtain the edge weight (i.e., the conditional association). The nodewise approach has been shown to reveal the true graph structure. However, for continuous variables, the regression coefficients are scaled differently than the partial correlations, and therefore the nodewise approach may lead to different edge weights. Here, the aggregation of the two regression coefficients is crucial in obtaining the true partial correlation. We show that when the correlations of the two predictors with the control variables are different, averaging the regression coefficients leads to an asymptotically biased estimator of the partial correlation. This is likely to occur when a variable has a high correlation with other nodes in the network (e.g., variables in the same domain) and a lower correlation with another node (e.g., variables in a different domain). We discuss two different ways of aggregating the regression weights, which can obtain the true partial correlation: first, multiplying the weights and taking their square root, and second, rescaling the regression weight by the residual variances. The two latter estimators can recover the true network structure and edge weights.

KEYWORDS
Network analysis; nodewise regression; partial correlation; regression coefficient; asymptotic properties

Psychometric network modeling has been widely used in psychological research to conceptualize phenomena as systems of interacting variables (Borsboom et al., 2021; Isvoranu et al., 2022). A psychometric network consists of nodes representing variables that make up the construct of interest (e.g., symptoms of a disorder, scores on a test, or items of a personality trait). The nodes are connected by edges that represent partial associations, that is the strength of the relation between two nodes after controlling for the influence of all other nodes. Edges are estimated with two approaches. The partial correlations can be obtained using the inverse of the covariance matrix (Epskamp et al., 2018; Waldorp & Marsman, 2022). In this case, the network edges are obtained using the *joint distribution* of all variables. This distribution cannot be obtained for all types of networks (e.g., mixed graphical models; Haslbeck & Waldorp, 2015) or it may be too computationally expensive to compute (Meinshausen & Bühlmann, 2006; van Borkulo et al., 2014;). Alternatively, a simpler method has been suggested that uses the *conditional distribution* of each node to approximate the joint distribution. These conditional distributions are obtained by regressing each node on all other nodes in a generalized linear regression model (Besag, 1975; van Borkulo et al., 2014). In this approach, called *nodewise regression*, two estimates are obtained for each edge (e.g., for edge $X—Y$: X is predicted by Y and all other nodes to obtain the regression weight $\beta_{XY.Z}$, and then Y is predicted by X and all other nodes to obtain the regression weight $\beta_{YX.Z}$). The two regression coefficients $\beta_{YX.Z}$ and $\beta_{XY.Z}$ are aggregated to obtain an approximation of the partial correlation of the $X—Y$ edge. In psychometric networks, the nodewise approach is especially used for graphical models of binary and mixed data as well as longitudinal estimation (e.g., see Epskamp et al., 2018; Haslbeck & Waldorp, 2015; van Borkulo et al., 2014). Nodewise regression is a useful tool for obtaining network parameters. It can consistently determine edge presence or absence and obtain the true network structure (Meinshausen & Bühlmann, 2006; Waldorp & Marsman, 2022).

In this note, we show that the true edge weights—the strength of the parameters—are not necessarily obtained with the nodewise approach. Rather, the method of aggregating the two regression parameters is crucial in ensuring an unbiased asymptotic estimator of the partial correlation. Bias is used in this paper to denote the difference between the *asymptotic values* of the partial correlation and the aggregated nodewise regression estimator. Simply averaging nodewise regression parameters leads to an asymptotically biased estimator of the non-zero partial correlation of multivariate normal variables. We highlight alternatives for aggregating the two regression parameters which allow the nodewise approach to accurately obtain the partial correlation: multiplying and rescaling the regression weights. The paper is organized as follows: First, we provide some background on partial correlations and regression coefficients. Second, we discuss averaging the two regression weights and show under what conditions the nodewise regression approach leads to biased or unbiased non-zero partial correlations. Third, we highlight alternative approaches to combining the two regression parameters. Finally, we extend the derivations to unstandardized and binary variables. We demonstrate the utility of the alternative nodewise approaches with an example of online learning readiness.

Background: partial correlation and regression coefficient

Partial correlations and regression coefficients are different statistics that describe the association between two variables of interest. In this section, we will consider three variables, X, Y, and Z, which we assume to be normally distributed and mean-centered with unit-variance. We start with the partial correlation $\rho_{YX.Z}$. The partial correlation is the correlation between two variables (e.g., X and Y) after having controlled for the influence of a third variable (i.e., Z). The partial correlation coefficient is defined as

$$\rho_{YX.Z} = \frac{\rho_{XY} - \rho_{XZ}\rho_{YZ}}{\sqrt{(1-\rho_{XZ}^2)(1-\rho_{YZ}^2)}}, \tag{1}$$

where ρ_{XY} denotes the marginal correlation between variables X and Y. Note that this equation is standardized by the total variance of X and Y that is not explained by the remaining variable (i.e., Z), that is, the terms $1-\rho_{XZ}^2$ and $1-\rho_{YZ}^2$ in the denominator. Therefore, the partial correlation is the correlation between the residuals of X predicted by Z and the residuals of Y predicted by Z. The partial correlation is bounded between -1 and 1.

In contrast, the regression coefficient $\beta_{YX.Z}$ is the weight obtained by regressing the dependent variable Y on X and Z. It can be obtained by

$$\beta_{YX.Z} = \frac{\rho_{XY} - \rho_{XZ}\rho_{YZ}}{(1-\rho_{XZ}^2)}. \tag{2}$$

In this equation, the denominator only controls for the unexplained variance of X (i.e., $1-\rho_{XZ}^2$). The regression weights are unbounded and can be interpreted as the expected change in Y when increasing X by one unit and keeping Z fixed.

The numerator of Equations 1 and 2 are equal; it is $\rho_{XY} - \rho_{YZ}\rho_{XZ}$. The numerators determines whether a parameter is zero, as no value of a non-zero denominator can produce a zero result. Thus, $\beta_{YX.Z} = 0$ if and only if $\rho_{YX.Z} = 0$. The denominator differs. The partial correlation coefficient is scaled by the unexplained variance of both X and Y, whereas the regression coefficient is scaled only by the unexplained variance of the predictor X. As a result of this difference in scaling, the partial correlation $\rho_{YX.Z}$ will be high if a large portion of the unexplained-by-Z portion of Y is captured by the residuals of X. In contrast, the regression coefficient $\beta_{YX.Z}$ will be high if the unexplained-by-Z portion of Y captures a large portion of the *total variance* of X. For example, if Z is highly correlated with both X and Y, the regression coefficient will have a relatively lower value as there is a small portion of the total variance in Y that remains unexplained by Z and can still be captured by X. The partial correlation may still be large as it quantifies the strength of the relationship between X and Y after removing the variance explained by Z from both variables.

The regression coefficients are zero if and only if the partial correlation is zero.[1] This allows the nodewise approach to consistently detect the true graph structure (Meinshausen & Bühlmann, 2006; Waldorp & Marsman, 2022). However, as we have also seen, the regression coefficient is a differently scaled partial correlation, and as such non-zero edges may differ. Here it becomes crucial how the two regression weights are aggregated to obtain the true edge weight, the partial correlation.

Averaging nodewise parameters is a biased estimator of the partial correlation

We first discuss aggregating the regression weights by merely averaging them. We make use of Equations 1

[1] Due to bias in the *estimation*, for example due to l_1 penalized estimation (Brusco et al., 2023), the parameter estimates may not necessarily both be zero, even if the true population values are equal.

and 2 to see when the averaged nodewise estimator is equal to the partial association.

$$\rho_{YX.Z} = \frac{(\beta_{YX.Z} + \beta_{XY.Z})}{2}$$
$$\frac{\rho_{XY} - \rho_{XZ}\rho_{YZ}}{\sqrt{(1-\rho_{XZ}^2)(1-\rho_{YZ}^2)}} = \frac{\rho_{XY} - \rho_{YZ}\rho_{XZ}}{2(1-\rho_{XZ}^2)} + \frac{\rho_{XY} - \rho_{XZ}\rho_{YZ}}{2(1-\rho_{YZ}^2)}$$
$$\frac{1}{\sqrt{(1-\rho_{XZ}^2)(1-\rho_{YZ}^2)}} = \frac{1}{2(1-\rho_{XZ}^2)} + \frac{1}{2(1-\rho_{YZ}^2)}$$
$$1 = \frac{\sqrt{(1-\rho_{XZ}^2)(1-\rho_{YZ}^2)}}{2(1-\rho_{XZ}^2)} + \frac{\sqrt{(1-\rho_{XZ}^2)(1-\rho_{YZ}^2)}}{2(1-\rho_{YZ}^2)}$$
$$1 = \frac{\sqrt{(1-\rho_{XZ}^2)(1-\rho_{YZ}^2)}(1-\rho_{YZ}^2) + \sqrt{(1-\rho_{XZ}^2)(1-\rho_{YZ}^2)}(1-\rho_{XZ}^2)}{2(1-\rho_{YZ}^2)(1-\rho_{XZ}^2)}$$
$$1 = \frac{(1-\rho_{YZ}^2) + (1-\rho_{XZ}^2)}{2\sqrt{(1-\rho_{YZ}^2)(1-\rho_{XZ}^2)}}$$
$$2\sqrt{(1-\rho_{YZ}^2)(1-\rho_{XZ}^2)} = (1-\rho_{YZ}^2) + (1-\rho_{XZ}^2)$$
$$0 = (\sqrt{(1-\rho_{YZ}^2)} - \sqrt{(1-\rho_{XZ}^2)})^2$$
$$\rho_{XZ}^2 = \rho_{YZ}^2$$

The partial correlation is well approximated by the averaged regression coefficients if the variance of X explained by Z (i.e., ρ_{XZ}^2) is equal to the variance of Y explained by Z (i.e., ρ_{YZ}^2), in other words, when both X and Y are equally well predicted by Z.

If the variance of X and Y explained by Z is not equal, what is the effect on the β's? If ρ_{XZ}^2 is smaller than ρ_{YZ}^2, then $1-\rho_{XZ}^2$ will be larger than $1-\rho_{YZ}^2$. The larger denominator will lead to a smaller regression coefficient and as a result $\beta_{YX.Z}$ will be smaller than $\beta_{XY.Z}$. The smaller denominator will lead to a larger regression coefficient, which can take values outside the range $[-1, 1]$. When $\beta_{YX.Z}$ and $\beta_{XY.Z}$ are averaged, the averaged nodewise edge parameter will be asymptotically biased and overestimate the true partial correlation. The difference in variances is likely to occur in practical applications, for example, when assessing exogenous variables, classical test-theoretical questionnaires, or a network assessing two domains. We review these practical applications in Appendix A.

How large is the bias of the averaged nodewise approach?

How does a difference in residual variance affect the ability of the averaged regression coefficient to capture the partial correlation? We assessed the bias by sequencing different combinations of positive correlations between the variables, namely: ρ_{XY}, ρ_{XZ}, and ρ_{YZ}.[2] In particular, we varied both ρ_{XY} and ρ_{YZ} from 0.1 to 0.7 in steps of 0.2. The remaining correlation ρ_{XZ} was sequenced in steps of 0.001 from the lowest to the highest bound of the correlation given by the other two correlations.[3] For each combination of correlation coefficients, we obtained the partial correlation $\rho_{YX.Z}$ and the regression coefficients $\beta_{XY.Z}$ and $\beta_{YX.Z}$ using Equation 1 and 2. Bias was defined as the absolute difference between the averaged regression coefficients $(\beta_{YX.Z} + \beta_{XY.Z})$ 2 and the partial correlation $\rho_{YX.Z}$.

Figure 1 shows the results of the bias. The averaged nodewise regression coefficient overestimates the strength of the partial correlation. Often, the bias is negligible. However, there are certain situations where the bias grows concerningly large; it can be as large as 0.6. This is particularly the case when ρ_{XY} and ρ_{YZ} are small and ρ_{XZ} moves further away from zero. Here, the difference between ρ_{XZ} and ρ_{YZ} becomes larger and, as expected from the derivations, the bias is largest.

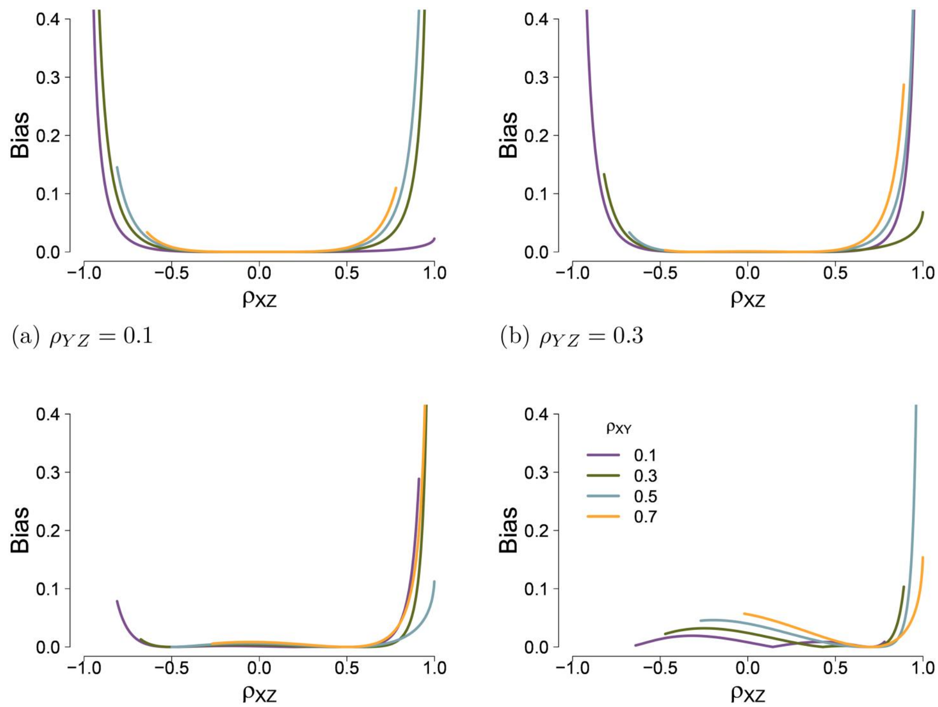
Extension to networks with more than three variables

In reality, most networks will have more than three nodes. In these cases, determining the difference in the two parameters becomes more complicated to compute because it depends on the correlation structure of a larger set of variables. Nevertheless, the bias essentially reduces to the same configuration as above, but instead of the marginal correlations, the bias depends on the values of the three respective partial correlations. Suppose there are three focal variables X, Y, and Z in the network, in addition to a set of k control variables (i.e., $\mathbf{C} = \{C_1, ..., C_k\}$). Note that Z is no different from and could be any of the k control variables in $\mathbf{C}$. Here, the correlation coefficients in the above equations are no longer ρ_{XY}, ρ_{YZ}, and ρ_{XZ}, but rather the partial correlations $\rho_{XY.\mathbf{C}}$, $\rho_{YZ.\mathbf{C}}$, and $\rho_{XZ.\mathbf{C}}$.

If the set of control variables $\mathbf{C}$ is independent of the set of focal variables, the above derivations remain unchanged. However, if $\mathbf{C}$ is correlated with the set of focal nodes, the bias may differ. Without loss of generality, consider the case where ρ_{XZ}^2 is larger than ρ_{YZ}^2. The bias is then *smaller* if $\mathbf{C}$ explains a substantial part of the larger marginal correlation coefficient ρ_{XZ}^2 without explaining a substantial proportion of the smaller coefficient ρ_{YZ}^2. This situation would result in the partial correlation $\rho_{XZ.\mathbf{C}}^2$ being smaller than the marginal correlation ρ_{XZ}^2, while the partial correlation

[2]The code to replicate the findings can be found at https://osf.io/gy8px/.

[3]For three variables, one can derive the bounds of a correlation if the two remaining correlations are fixed and known. The formula for the upper bound is $\rho_{XZ} \leq \rho_{XY}\rho_{YZ} + \sqrt{(1-\rho_{XZ}^2)(1-\rho_{YZ}^2)}$ and for the lower bound $\rho_{XZ} \geq \rho_{XY}\rho_{YZ} - \sqrt{(1-\rho_{XZ}^2)(1-\rho_{YZ}^2)}$.



(a) $\rho_{YZ} = 0.1$ (b) $\rho_{YZ} = 0.3$

(c) $\rho_{YZ} = 0.5$ (d) $\rho_{YZ} = 0.7$

Figure 1. Absolute bias in $(\beta_{YX.Z} + \beta_{XY.Z})/2$ relative to $\rho_{YX.Z}$. *Note.* Each quadrant reflects one setting of the correlation ρ_{YZ} and each line within all the plots reflects a different specification of the correlation ρ_{XY}.

$\rho^2_{YZ.\mathbf{C}}$ remains the same as its corresponding marginal correlation ρ^2_{YZ}. As such, the difference between the two partial correlation coefficients is smaller than the difference between the two marginal correlations and the bias could decrease to a negligible one. The bias becomes *larger* if **C** increases the difference in the partial associations. For example, this would happen if **C** explained a portion of the smaller coefficient ρ^2_{YZ} but did not affect the larger coefficient ρ^2_{XZ}. Here, the difference between the partial correlations would be larger, $\rho^2_{XZ.\mathbf{C}}$ would be even larger than $\rho^2_{YZ.\mathbf{C}}$, and so would be the bias. Similarly, if instead we considered ρ^2_{XZ} and ρ^2_{YZ} to be equal, but **C** explained a large part of only one of the two coefficients, say ρ^2_{XZ}, the partial correlation $\rho^2_{XZ.\mathbf{C}}$ would be relatively smaller than $\rho^2_{YZ.\mathbf{C}}$, introducing bias into the estimator.

Unbiased nodewise estimators with multiplied or rescaled aggregation of regression parameters

Nodewise parameters can also be aggregated in other ways, such as by multiplying or rescaling the weights (Epskamp et al., 2018; Krämer et al., 2009).

Multiplying regression weights

The nodewise estimator can also be aggregated by multiplying the two regression parameters and taking the square root of the multiplied coefficients (Krämer et al., 2009). The multiplied nodewise estimator is calculated as

$$\rho_{YX.Z} = \text{sgn}(\beta_{YX.Z})\sqrt{\beta_{YX.Z}\beta_{XY.Z}}$$

$$\frac{\rho_{XY} - \rho_{XZ}\rho_{YZ}}{\sqrt{(1-\rho^2_{XZ})(1-\rho^2_{YZ})}} = \text{sgn}(\beta_{YX.Z})\sqrt{\frac{\rho_{XY} - \rho_{YZ}\rho_{XZ}}{(1-\rho^2_{XZ})}\frac{\rho_{XY} - \rho_{XZ}\rho_{YZ}}{(1-\rho^2_{YZ})}}$$

$$= \text{sgn}(\beta_{YX.Z})\frac{\rho_{XY} - \rho_{XZ}\rho_{YZ}}{\sqrt{(1-\rho^2_{XZ})(1-\rho^2_{YZ})}}$$

where $\text{sgn}(\cdot)$ is the sign function, which is $+1$ for positive values and -1 for negative values. The multiplied estimator equates to the partial correlation estimator. As such, the aggregation leads to an asymptotically unbiased estimator of the partial correlation using nodewise regression. The sign function ensures the correct sign of the multiplied nodewise parameter.

Rescaling regression parameters

Alternatively, one can also choose to rescale the regression parameter with the residual variances (Epskamp et al., 2018). The rescaled parameter is calculated as:

$$\rho_{YX.Z} = \beta_{YX.Z}\frac{\sqrt{(1-\rho^2_{XZ})}}{\sqrt{(1-\rho^2_{YZ})}}$$

$$\frac{\rho_{XY} - \rho_{XZ}\rho_{YZ}}{\sqrt{(1-\rho^2_{XZ})(1-\rho^2_{YZ})}} = \frac{\rho_{XY} - \rho_{YZ}\rho_{XZ}}{(1-\rho^2_{XZ})}\frac{\sqrt{(1-\rho^2_{XZ})}}{\sqrt{(1-\rho^2_{YZ})}}$$

$$= \frac{\rho_{XY} - \rho_{XZ}\rho_{YZ}}{\sqrt{(1-\rho^2_{XZ})(1-\rho^2_{YZ})}}$$

where one can equally use the second regression parameter $\beta_{XY.Z}$ instead by also switching the numerator and denominator of the fraction of residual variances. The rescaled nodewise estimator equates to the partial correlation and is also an asymptotically unbiased estimator of the partial correlation.

Nodewise aggregation in unstandardized and binary variables

Until now, we assumed the variables X, Y, and Z to be mean-centered with unit-variance and multivariate normally distributed. How does the nodewise approach behave for unstandardized or binary variables? For unstandardized variables, the averaged nodewise estimator is asymptotically biased also (see Appendix B). Unstandardized variables likely increase the chance of asymptotic bias because equality of the variance on top of the covariances must be satisfied to ensure no asymptotic bias.

For binary variables, a different pattern occurs. The network model and type of nodewise regression changes to an Ising model (Ising, 1925) and logistic regression respectively (Keetelaar et al., 2024; van Borkulo et al., 2014). Both the Ising model's partial associations and the nodewise approach's regression weights consist of a transformed odds ratio. In fact, the nodewise estimator is equal to the Ising parameter, and as such the nodewise approach is asymptotically *unbiased* for binary variables (see Appendix C). The undesirable properties of the averaged nodewise estimator for multivariate normally distributed variables are not exhibited by the nodewise approach for binary variables.

Example application of the nodewise estimators

We illustrate the performance of the three aggregated nodewise estimators in an empirical application with a dataset assessing university students' readiness for E-learning (Nguyen et al., 2022). The dataset was originally published as a network that included six domains measured with 33 items, comprising computer skills, internet skills, online communication, self-learning, self-control, and online motivation.[4] Higher scores on the domains indicate that students are better prepared to learn online during the COVID-19 pandemic. In our example, we re-analysed the dataset with the joint approach, as well as the averaged, multiplied, and rescaled nodewise estimator. The final sample included 1,377 participants. All analyses were conducted in R (R-Core-Team, 2024), using the packages **ppcor** for partial correlation estimation (Kim, 2015), **tidyverse** for comprehensive data manipulation and analysis (Wickham, 2017), and **qgraph** for the visualization of the networks (Epskamp et al., 2012).

Figure 2 depicts the results of the analysis. In the network obtained with the joint approach, self-control is strongly connected to self-learning ($r_{xy.z} = 0.52$) and online motivation ($r_{xy.z} = 0.44$). Self-learning and online motivation are connected but show a slightly weaker link ($r_{xy.z} = 0.2$). The three domains show weak links with the skill domains; here the strongest connection is between online motivation and online communication ($r_{xy.z} = 0.16$). The skill domains are strongly connected depicting partial correlations up to 0.5.

There was a noticeable difference between the estimates of the joint and averaged nodewise approach (see Figure 2c). In particular, the edges between self-learning and self-control as well as between self-control and online motivation were estimated to be stronger in the averaged nodewise regression, with differences of 0.16 and 0.14 respectively. The edges between the skills variables and the personality variables were estimated with a bias of up to 0.02. The skills variables showed a slight bias between online communication and internet skills (i.e., 0.02) and between computer skills and internet skills (i.e., 0.03). In contrast, the multiplied and rescaled nodewise estimators were able to retrieve the partial correlations of the joint approach exactly.

In sum, in this example, the averaged nodewise regression led to a non-negligible bias of edge weight estimates whereas the multiplied and rescaled nodewise estimators were able to retrieve the partial correlation, as expected.

Conclusions

Performing nodewise regression to obtain the network model is a powerful alternative to the potentially cumbersome and computationally intensive alternative of joint modeling. It allows one to reliably obtain the graph structure (Meinshausen & Bühlmann, 2006; Waldorp & Marsman, 2022). However, the nodewise approach does not always detect the true edge weights, its goodness depends on the aggregation of the two regression parameters. Averaging the two regression weights leads to an asymptotically biased estimator of the partial correlation when two variables are strongly

[4]The data can be downloaded alongside the original publication https://jeehp.org/journal/view.php?doi=10.3352/jeehp.2022.19.22 and relevant R-code for the analysis can be found on OSF https://osf.io/gy8px/.

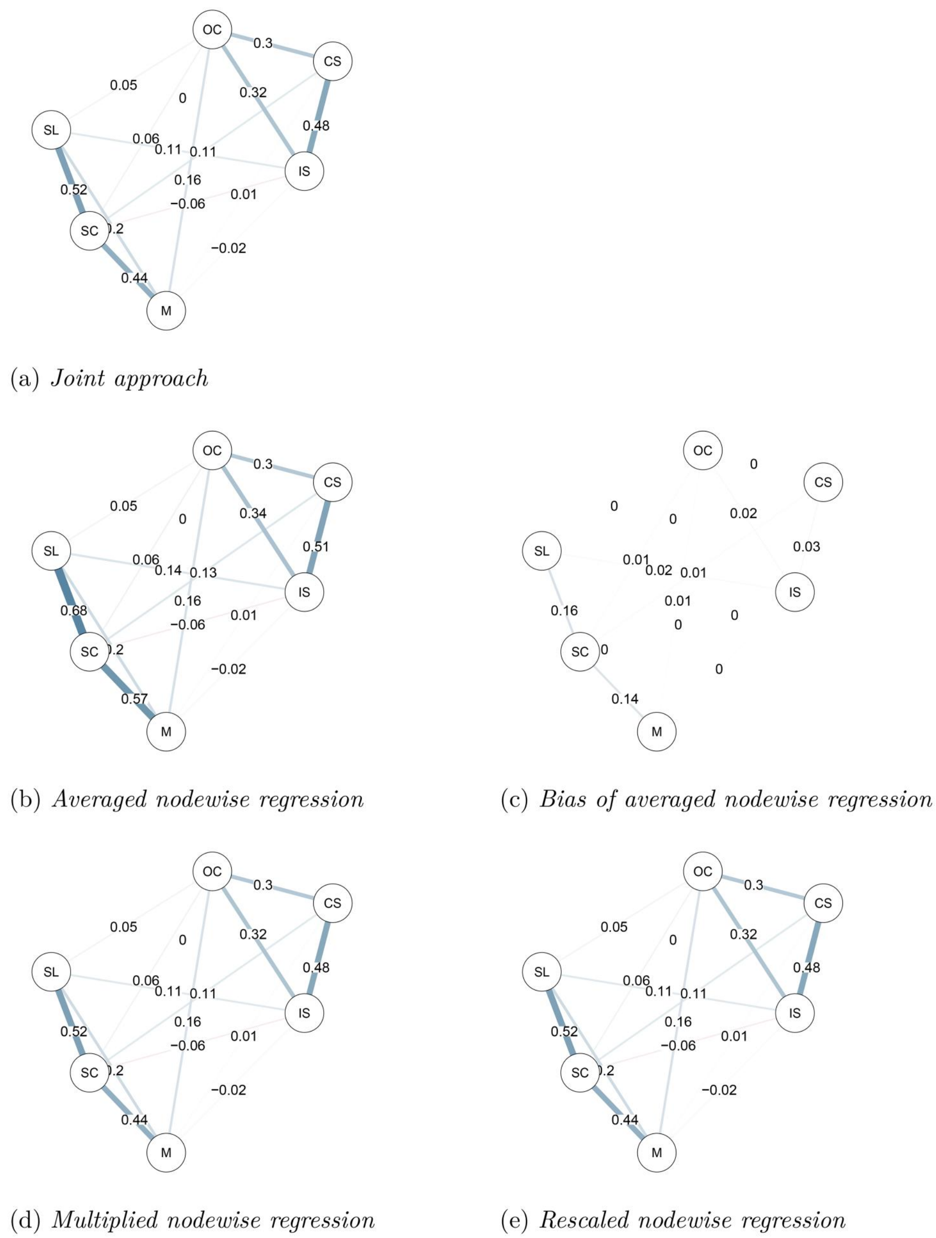


Figure 2. Networks obtained through the different estimation approaches. *Note*. Networks resulting from the joint estimation (top), the averaged nodewise approach (middle left), the difference between both (middle right), the multiplied nodewise approach (bottom left), and the rescaled nodewise approach (bottom right). Blue edges indicate positive values and red edges indicate negative ones; edge thickness reflects association strength. CS: Computer Skills, IS: Internet Skills, OC: Online Communication, SL: Self-Learning, SC: Self-Control, M: Online Motivation.

correlated and weakly correlated with other nodes in the network. There are two alternative ways of aggregating the regression parameters which are asymptotically unbiased estimators: first, by multiplying the coefficients and taking the square root, and second, by rescaling one regression parameter with the residual variances. The multiplied estimator might have a slight advantage, as it can be derived directly from the matrix of regression weights. In contrast, the rescaled estimator requires only one of the regression weights but it also depends on the residual variances.

Our work did not evaluate how the estimators perform in finite samples or biased estimation approaches. For example, for l_1 regression, one can expect worse performance of the disjoint estimator even when equality holds asymptotically, as is seen in the Ising model (Brusco et al., 2023). When researchers use ordinary least squares or maximum likelihood estimation, asymptotic equality should hold in the estimated parameters. However, future work should evaluate their properties in a simulation study.

In conclusion, the multiplied and rescaled nodewise estimators have desirable asymptotic properties. They show clear advantages over the averaged nodewise estimator.

Article information

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Appendix A: When will bias arise in applications?

The averaged nodewise coefficient will be biased if the correlation of the two variables of interest with a third variable is different. We see four applications where one might expect a difference in correlations and therefore an increased risk of bias.

1. *Exogenous variables:* Researchers often include exogenous variables in the network model, that is, variables that are external to the construct being studied. Examples of such variables are sociodemographic factors (e.g., age, ethnicity, sex) and treatment allocation. We expect the system variables to be highly correlated with each other (e.g., depressed mood is highly correlated with one's self-esteem) but less correlated with the exogenous variables (e.g., ethnicity is less strongly correlated with depressed mood). As such, we would expect the edge weights between the exogenous variables and the system variables to be biased.
2. *Classic test-theoretical questionnaire:* In classical psychological scale development, a questionnaire is designed to be maximally reliable. To achieve that aim, often several items are included that measure the same or very close entities. This could be a simple rewording or a reverse wording of an item; these two items tend to be highly correlated with each other, but less correlated with the other items in the questionnaire. If modeling the full questionnaire, bias is expected in the edges between the almost identical items and all other questionnaire items.
3. *Network of two domains:* Comorbidity research using the network approach examines interactions between symptoms of two psychopathologies. Symptoms of both psychopathologies are included in the same network, and interactions between them are examined for potential bridge symptoms. If symptoms of the same psychopathology are more strongly correlated with each other than with symptoms of other psychopathologies, the associations between disorders—the associations of the bridge symptoms—are at risk for biased estimates.
4. *Clustering research:* A network cluster is a group of nodes that are more strongly connected to nodes within the same cluster than to nodes from other clusters. By definition, one assumes a difference in correlation strength between different nodes, where the correlations within clusters are higher than between clusters. This pattern of correlations could introduce bias in the edges that link the different clusters.

These are just a few examples of applications where differences in correlations, and thus bias, might be expected. Of course, bias is not limited to these situations and does not necessarily occur in them.

Appendix B: Derivations for unstandardized variables

In the main manuscript, we assumed the variables X, Y, and Z to be mean-centered with unit-variance and multivariate normally distributed. How does the nodewise approach behave outside these assumptions, such as for unstandardized variables?

For unstandardized variables, the partial correlation is of the form

$$\rho_{YX.Z} = \frac{\sigma_{XY}\sigma_Z^2 - \sigma_{XZ}\sigma_{YZ}}{\sqrt{\sigma_X^2\sigma_Z^2 - \sigma_{XZ}^2}\sqrt{\sigma_Y^2\sigma_Z^2 - \sigma_{YZ}^2}}$$

and the regression weight of the form

$$\beta_{YX.Z} = \frac{\sigma_X(\sigma_{XY}\sigma_Z^2 - \sigma_{XZ}\sigma_{YZ})}{\sigma_Y(\sigma_X^2\sigma_Z^2 - \sigma_{XZ}^2)}.$$

Contrary to the standardized form, the numerators differ: the regression coefficient contains an additional σ_X-term. As in the standardized case, the denominator differs: Whereas the partial correlation is scaled by the residuals of both X and Y, the regression weight is now scaled by the residual of X and the *total variance* of Y. How does this difference in scaling affect the ability of the nodewise approach to approximate the partial correlation in unstandardized variables?

Averaging nodewise estimator in unstandardized variables

For the averaged nodewise estimator, the approximation is equal for unstandardized variables if

$$\rho_{YX.Z} = \frac{(\beta_{YX.Z} + \beta_{XY.Z})}{2}$$
$$\frac{\sigma_{XY}\sigma_Z^2 - \sigma_{XZ}\sigma_{YZ}}{\sqrt{\sigma_X^2\sigma_Z^2 - \sigma_{XZ}^2}\sqrt{\sigma_Y^2\sigma_Z^2 - \sigma_{YZ}^2}} = \frac{\sigma_X(\sigma_{XY}\sigma_Z^2 - \sigma_{XZ}\sigma_{YZ})}{2\sigma_Y(\sigma_X^2\sigma_Z^2 - \sigma_{XZ}^2)} + \frac{\sigma_Y(\sigma_{XY}\sigma_Z^2 - \sigma_{XZ}\sigma_{YZ})}{2\sigma_X(\sigma_Y^2\sigma_Z^2 - \sigma_{YZ}^2)}$$

or in simpler terms, if $\sigma_X = \sigma_Y$ as well as $\sigma_{XZ}^2 = \sigma_{YZ}^2$. In fact, the nodewise regression approximation with unstandardized variables reduces to that of the standardized variables if $\sigma_X = \sigma_Y$. When this equality holds, the nodewise approach obtains the true network structure, but again produces biased edge weights unless $\sigma_{XZ}^2 = \sigma_{YZ}^2$.

Multiplied and rescaled nodewise aggregation in unstandardized variables

If instead of averaging the nodewise estimates to obtain the partial correlation, one aggregates the estimates by multiplying the variables, the derivations are as follows

$$\begin{aligned}
&\frac{\sigma_{XY}\sigma_Z^2 - \sigma_{XZ}\sigma_{YZ}}{\sqrt{\sigma_X^2\sigma_Z^2 - \sigma_{XZ}^2}\sqrt{\sigma_Y^2\sigma_Z^2 - \sigma_{YZ}^2}} \\
&= \text{sgn}(\beta_{YX.Z})\sqrt{\frac{\sigma_X(\sigma_{XY}\sigma_Z^2 - \sigma_{XZ}\sigma_{YZ})}{\sigma_Y(\sigma_X^2\sigma_Z^2 - \sigma_{XZ}^2)}\frac{\sigma_Y(\sigma_{XY}\sigma_Z^2 - \sigma_{XZ}\sigma_{YZ})}{\sigma_X(\sigma_Y^2\sigma_Z^2 - \sigma_{YZ}^2)}} \\
&= \text{sgn}(\beta_{YX.Z})\frac{\sigma_{XY}\sigma_Z^2 - \sigma_{XZ}\sigma_{YZ}}{\sqrt{(\sigma_X^2\sigma_Z^2 - \sigma_{XZ}^2)}\sqrt{(\sigma_Y^2\sigma_Z^2 - \sigma_{YZ}^2)}},
\end{aligned}$$

where $\text{sgn}(\cdot)$ is the sign function, which is $+1$ for positive values and -1 for negative values. Respectively, the derivations for the rescaled nodewise estimator are as follows

$$\frac{\sigma_{XY}\sigma_Z^2-\sigma_{XZ}\sigma_{YZ}}{\sqrt{\sigma_X^2\sigma_Z^2-\sigma_{XZ}^2}\sqrt{\sigma_Y^2\sigma_Z^2-\sigma_{YZ}^2}}$$
$$=\frac{\sigma_X(\sigma_{XY}\sigma_Z^2-\sigma_{XZ}\sigma_{YZ})}{\sigma_Y(\sigma_X^2\sigma_Z^2-\sigma_{XZ}^2)}\frac{\sqrt{(\sigma_X^2\sigma_Z^2-\sigma_{XZ}^2)}}{\sqrt{(\sigma_Y^2\sigma_Z^2-\sigma_{YZ}^2)}}$$
$$=\frac{\sigma_{XY}\sigma_Z^2-\sigma_{XZ}\sigma_{YZ}}{\sqrt{\sigma_X^2\sigma_Z^2-\sigma_{XZ}^2}\sqrt{\sigma_Y^2\sigma_Z^2-\sigma_{YZ}^2}}.$$

As such, both the rescaled and multiplied estimators are asymptotically unbiased also for unstandardized variables.

Appendix C: Nodewise approach is asymptotically unbiased for binary variables

For simplicity, we assume three variables X, Y, and Z to take on a value in the binary set {0, 1}. Z here is merely a control variable and we are trying to estimate the partial association between X and Y. If X takes on the value 0, we denote it x, and if it takes on the value 1 we denote it $\bar{x}$. This also holds respectively for both Y and Z.

Ising model

First, we derive the edge association parameter for the Ising model. The full Ising model is formulated as

$$p(\mathbf{X})=\frac{1}{N}\exp\left(\sum_{i=1}^{p}x_i\mu_i+2\sum_{i=1}^{p-1}\sum_{j=i+1}^{p}x_ix_j\sigma_{ij}\right),$$

where μ_i represents the threshold parameter for variable i, σ_{ij} the pairwise interaction parameter between variables i and j, and N the normalizing constant. For the three variables X, Y, and Z, the Ising model is described by

$$p(X,Y,Z)=\frac{1}{N}\exp\left(X\mu_X+Y\mu_Y+2XY\sigma_{XY}+Z\mu_Z+2XZ\sigma_{XZ}+2YZ\sigma_{YZ}\right)$$

To derive the pairwise interaction parameter, we divide the odds of X by the odds of Y.

$$\frac{p(x,y,Z)}{p(\bar{x},y,Z)}\frac{p(\bar{x},\bar{y},Z)}{p(x,\bar{y},Z)}$$

The normalizing constant cancels out completely and the odds ratio becomes

$$\frac{\dfrac{\exp(\mu_Z Z)}{\exp(\mu_X+\mu_Z Z+2Z\sigma_{XZ})}}{\dfrac{\exp(\mu_X+\mu_Y+2\sigma_{XY}+\mu_Z Z+2Z\sigma_{XZ}+2Z\sigma_{YZ})}{\exp(\mu_Y+\mu_Z Z+2Z\sigma_{YZ})}}$$
$$=\exp(2\sigma_{XY})$$

Therefore, the pairwise association is defined by the probabilities

$$\exp(2\sigma_{XY})=\frac{p(x,y,Z)}{p(\bar{x},y,Z)}\frac{p(\bar{x},\bar{y},Z)}{p(x,\bar{y},Z)}$$

Logistic regression

A logistic regression is of the form

$$\frac{p(\bar{y}|X,Z)}{p(y|X,Z)}=\exp(\beta_0+\beta_X X+\beta_Z Z).$$

The regression coefficient β_X is the odds of Y with a one-unit change of X

$$\exp(\beta_X)=\frac{\exp(\beta_0+\beta_X(X+1)+\beta_Z Z)}{\exp(\beta_0+\beta_X X+\beta_Z Z)}$$

By inserting the odds for the logistic equation the regression parameter is defined as

$$\exp(\beta_X)=\frac{p(\bar{y}|\bar{x},Z)}{p(y|\bar{x},Z)}\bigg/\frac{p(\bar{y}|x,Z)}{p(y|x,Z)}=\frac{p(\bar{y}|\bar{x},Z)p(y|x,Z)}{p(y|\bar{x},Z)p(\bar{y}|x,Z)}$$

Similarly, the regression coefficient β_Y is the odds of X with a one-unit change in Y

$$\frac{p(\bar{x}|Y,Z)}{p(x|Y,Z)}=\exp(\beta_0+\beta_Y Y+\beta_Z Z)$$

$$\exp(\beta_Y)=\frac{\exp(\beta_0+\beta_Y(Y+1)+\beta_Z Z)}{\exp(\beta_0+\beta_Y Y+\beta_Z Z)}$$

$$\exp(\beta_Y)=\frac{p(\bar{x}|\bar{y},Z)}{p(x|\bar{y},Z)}\bigg/\frac{p(\bar{x}|y,Z)}{p(x|y,Z)}=\frac{p(\bar{x}|\bar{y},Z)p(x|y,Z)}{p(x|\bar{y},Z)p(\bar{x}|y,Z)}$$

The two regression coefficients look different at first. For β_X we condition on X, whereas for β_Y, we condition on Y. However, we can rewrite the regression coefficients in terms of the joint probabilities rather than the conditional probabilities. Here we use the fact that

$$p(x|y,Z)=\frac{p(x,y,Z)}{p(y,Z)}$$

When replacing the conditional probabilities with all the respective joint probabilities, we get the following equation for β_X

$$\exp(\beta_X)=\frac{p(\bar{x},Z)p(\bar{y},\bar{x},Z)p(x,Z)p(y,x,Z)}{p(y,\bar{x},Z)p(\bar{x},Z)p(\bar{y},x,Z)p(x,Z)}$$
$$=\frac{p(\bar{y},\bar{x},Z)p(y,x,Z)}{p(y,\bar{x},Z)p(\bar{y},x,Z)}$$

and respectively for β_Y

$$\exp(\beta_Y)=\frac{p(\bar{y},Z)p(\bar{x},\bar{y},Z)p(y,Z)p(x,y,Z)}{p(x,\bar{y},Z)p(\bar{y},Z)p(\bar{x},y,Z)p(y,Z)}$$
$$=\frac{p(\bar{x},\bar{y},Z)p(x,y,Z)}{p(x,\bar{y},Z)p(\bar{x},y,Z)}$$

Therefore, β_X and β_Y are equal and their average is consistent with the Ising association parameter. As such, the averaged nodewise approach is an asymptotically unbiased estimator of the Ising model edge parameter.