

# Using Conditional Entropy Networks of Ordinal Measures to Examine Changes in Self-Worth Among Adolescent Students in High School

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## ABSTRACT

We implement an analytic approach for ordinal measures and we use it to investigate the structure and the changes over time of self-worth in a sample of adolescents students in high school. We represent the variations in self-worth and its various sub-domains using entropy-based measures that capture the observed uncertainty. We then study the evolution of the entropy across four time points throughout a semester of high school. Our analytic approach yields information about the configuration of the various dimensions of the self together with time-related changes and associations among these dimensions. We represent the results using a network that depicts self-worth changes over time. This approach also identifies groups of adolescent students who show different patterns of associations, thus emphasizing the need to consider heterogeneity in the data.

## KEYWORDS

Categorical data analysis;  
entropy networks;  
Longitudinal modeling;  
self-perceptions

## Introduction

The “self” is a long-standing construct studied within developmental and social psychology (Dweck, 1999; Harter, 1999, 2012). Self-evaluations connote individuals’ feelings of worthiness, competence, and significance both as a person in general (global self-worth) and in specific domains of competence or adequacy (domain-specific self-evaluations). The importance of self-evaluations lies in their robust and powerful effect upon emotions, motivational orientations, and achievement behaviors. Individuals holding positive self-worth evidence pride, confidence, and happiness, as well as choose optimal challenges and persist in achievement contexts. In contrast, those with lower self-worth may be prone toward depression and other negative effects, engage in unrealistic goal choices, and lack resiliency following achievement setbacks (Harter, 1999). These potential consequences of self-evaluations have led theorists and researchers toward concerted attempts to better understand the determinants, mediators, and consequences of global and domain-specific self-worth. Throughout the manuscript, we use the terms self-evaluations and self-worth to be consistent with Harter’s theory and measures.

When using a different term, we give the corresponding reference to avoid confusion.

Both the structure and content of self-evaluations have been the focus of several theories and empirical investigations. Structure refers to the underlying nature of how self-evaluations are conceptualized, such as unidimensional, multidimensional, and hierarchical structures (Fox, 1998; Marsh, 1990; Shavelson & Marsh, 1986). The older view that self-worth is an additive function of self-evaluations in multiple domains (unidimensional) paved the way toward contemporary views in which domain-specific self-evaluations contribute varying weight to formation of self-views (multidimensional), and that situation-specific self-evaluations that individuals experience on a shorter-time scale precede contextual, domain, and overall self-worth (hierarchical). The content of self-evaluations refers to those competence or adequacy domains that are salient to particular developmental levels (Harter, 1999; Marsh, 1990). For example, young children cannot distinguish a global sense of self but do discern self-evaluations in academic, physical, and social domains. During adolescence, domains of job, close friendship, romantic relationships, and morality emerge as areas in which self-evaluations

are drawn. Still more domains are differentiated during college, early/middle adulthood, and late adulthood. Thus both structure and content of global and domain-specific self-evaluations are important to consider in empirical inquiries of the self.

The approaches for studying “the self” *via* the uni-dimensional, single score approach, exemplified by the prevailing models and instruments popularized *via* the work of Coppersmith (1967) and Piers and Harris (1964) in the late 60’s and 70’s. These models were based on the assumption that the self was a unitary construct, best assessed by tapping a range of content, for example, how a child felt with peers, parents, in school, and that these evaluations could be summed into an overall evaluation of one’s general sense of self. This single score that represents one’s “general self-concept” could then be related to a variety of other constructs, outcomes, or indicators of well-being of interest to the investigator.

Rosenberg (1979) proposed an alternative approach that focused on global self-esteem as the target of measurement. He did not dispute the fact that people evaluate themselves differently in different domains of their lives. However, he claimed that these discriminations were difficult to assess accurately. Rather, an overall assessment of one’s worth as a person, in the form of a global judgment of self-esteem, would be sufficient to address as a predictor of other important life outcomes.

One particularly appealing and empirically tested model of self-evaluations is Harter’s (Harter, 1988, 1990, 1999) model of the causes, correlates, and consequences of global self-worth. The model embraces a multidimensional structure of global self-worth and is sensitive to developmental variations in the content of self-evaluations. The model is premised upon historical roots in the writings of James, Cooley, Mead, and Baldwin, and as such, views the self as both a cognitive and social construction. From a cognitive stance, global self-worth is determined by perceptions of competence or adequacy in domains viewed as important to an individual. This view is a central premise of contingencies of self-worth theory, based on which people pursue self-esteem by attempting to validate their abilities and qualities (Crocker & Wolfe, 2001). Individuals who value academic competence, for example, and view themselves as intelligent will evidence positive changes in global self-worth. In contrast, individuals who place importance on being physically attractive or skilled but view themselves as lacking in these areas will show lower self-worth. This view is in line with theories positing the need for communion and agency as two

fundamental dimensions of social cognition (Abele & Wojciszke, 2007, 2014; Fiske et al., 2017; Judd et al., 2005). Communion develops when trying to integrate the self into a larger social unit. Agency, in turn, develops from striving to expand a unique self and comprises qualities, such as efficiency and competence (Abele & Wojciszke, 2014).

Global self-worth is also socially determined in the reflected appraisals or perceptions of approval support by significant adults and peers. For example, according to the sociometer model (Leary & Baesister, 2000), self-esteem is a gauge of perceived social value that changes depending on how valued one feels by others. Individuals who perceive that important others view them in a favorable light will, in turn, feel satisfied and happy with themselves. Moreover, Harter specified two key correlates or consequences of global self-worth—*affective reactions and motivation*. Her original model (Harter, 1988) posited that global self-worth is accompanied by emotions, such as pride and shame, and that such affective feelings mediate the influence of global self-worth on motivated behaviors. In her most recent conceptualization (Harter, 1999), affects, such as depression and hopelessness are seen as correlates or even antecedents of global self-evaluations. In sum, Harter’s model of global self-worth identifies two primary determinants of self-worth (domain-specific self-conceptions and approval support) and two correlates or consequences (affect and motivation).

Several studies have focused upon perceptions of competence, particularly physical self-evaluations, as correlates or predictors of global self-worth. Several other studies have investigated developmental trends in level, accuracy, and sources of perceived physical competence, and their relation to affective and motivational variables (Weiss & Ferrer, 2002). These studies have contributed significantly to the knowledge base on physical self-evaluations. However, as Harter’s (Harter, 1988, 1999) model suggests, domain-specific self-conceptions comprise only one piece of the puzzle—the cognitive component—in explaining global self-worth. Perceived regard or approval support from parents, teachers, classmates, and close friends represent an important social component that, when added to perceived competence, may help explain more of the variance in global self-worth, as well as affective reactions and motivation in both academic and physical contexts.

Multiple longitudinal studies have examined changes in many of these self-evaluations over time. For example, self-esteem tends to decrease during early adolescence and then increase later on during

this period (Baldwin & Hoffmann, 2002). However, self-esteem seems to remain relatively stable over time across individuals, such as those with lower levels at one point are likely to show lower levels later on, relative to others (Robins & Trzesniewski, 2005). Importantly, changes in the various components of the self (or domain-specific self-esteem) have also been examined from childhood to adulthood (Orth et al., 2021). Some studies have shown that both global and domain-specific self-esteem increases from early adolescence to adulthood (von Soest et al., 2016). A meta-analysis (Orth et al., 2021) showed positive developmental trends in some of these components (e.g., academic ability, romantic relationships) and negative trends in other components (e.g., mathematics, morality). Similarly, a study compiling the literature, including meta-analyses, indicated that self-esteem is beneficial in all important life domains, such as work, relationships, or health, and that such benefits hold across age, gender, and race/ethnicity (Orth & Robins, 2022).

These longitudinal studies are important because they show developmental trends and associations between self-worth and relevant outcomes. What is lacking in the literature, however, is information about whether the various components that comprise the “self” change together over time. For example, it is important to understand if the theoretically described as antecedents of self-worth, such as perceived competence or perceived appearance indeed precede changes in self-worth, or if all these constructs change jointly as part of a unified system. Some studies have addressed this question focusing on a few of those components, primarily perceived competence, physical self-worth, and motivation (Ferrer & Gonzales, 2014; Ferrer & McArdle, 2003). They have shown that the changes in these domains are indeed interrelated over time. For example, when assessing the associations between perceived competence and motivation over time, motivation was found to be a leading indicator of changes in perceived competence, but not the other way around (Ferrer & McArdle, 2003). When physical self-worth was also considered, the patterns of associations were more complex. Changes in perceived self-worth were related to previous levels of self-worth and motivation, as well as to previous changes in perceived competence (Ferrer & Gonzales, 2014). These results provide support for the need of studying changes in the various components of self-worth jointly. Furthermore, if the model of the “self” is comprised of antecedents and consequences, these should be considered to

understand the mechanisms underlying changes in self-worth over time.

The goal of the present paper is to show how patterns of self-worth can be examined through the use of entropy measures. Given the ordinal nature of the data, we use a new analytic technique for ordinal data together with entropy measures derived from information theory. In particular, we use entropy-based measures to evaluate changes in the associations among the self-worth domains, as students go through a semester of high school, and we then use the networks representation to present the results. This approach allows us to investigate how the dependence between the various components of self-worth changes and evolves. An important novelty of this paper resides in the use of measures based on entropy to show how the observed uncertainty in global self-worth can be broken down as a function of several covariates, representing sub-domains. Using this approach allows us to describe such variability without making assumptions on the shape of the relationships between variables or distributional properties of the data, as it is the case in standard techniques, such as factor analysis.

Given that we establish directional associations from all the various responses of the covariates to the selected outcome, global self-worth, and that we represent the results using networks, our approach can be viewed from a psychometric networks perspective (Borsboom et al., 2021; Epskamp, 2020). Although a comparison with standard psychometric networks is not a goal of our paper, in the Discussion section we describe possible associations between these approaches.

## Methods

In this paper we use data from the “Motivation in High School Project,” a broader project aimed at examining self-perceptions and motivation among high school students (Ferrer & McArdle, 2003). The sample comprised 440 high school students (208 males and 221 females) ranging in age from 14 to 18 years (mean = 14.4,  $SD = 0.84$ ). On four occasions, starting the first week of the fall semester and every six weeks since then, students completed a questionnaire containing the Self-Perception Profile for Adolescents (Harter, 1988) in addition to measures of enjoyment, physical self-worth, and motivation. Together, the questionnaire included 50 items divided into 11 domains, namely: (a) perceived competence (*pc*), (b) physical appearance (*pa*), (c) physical self-worth (*psw*), (d) global self-worth (*gsw*), (e) parent support (*ps*), (f) teacher support (*ts*), (g) close-friend

support (*cfs*), (h) class-mate support (*cms*), (i) motivation (*mot*), (j) enjoyment (*enj*), and (k) self-determination (*sd*). Each domain was assessed with a Likert scale consisting of three to five items, each one with values ranging from 1 to 4. One sample item for each of the 11 Likert scales, which correspond to the 11 domains, is shown in Table A1 of Appendix A as an illustration.

### Data manipulation

Data based on Likert scales, such as those used in this study, are called *ordinal* data. Ordinal data is a type of categorical data that represents categories with a clear order allowing for a meaningful sequence. This is the case of self-rating data, like in our case, when people are asked to which degree they agree or disagree with a statement. These data are often analyzed using methodologies developed for continuous data (Agresti, 2003; Goodman, 1978; Liddell & Kruschke, 2018), leading to the use of measures and models conceived for the study of associations among interval variables, including the use of correlation and regression models. Ordinal data, however, cannot be naturally treated as having a continuous metric. Although the order between responses indicates some hierarchy (e.g., 4 is >3, and so on), the distance between these response options cannot be assumed to be the same across all responses, nor is the respondents' perception of the possible responses. The values used in these scales represent an ordered underlying concept, with the numbers serving merely as labels for that concept. In this context, using measures based on continuous variables not only is inappropriate, but it may also lead to erroneous results (Liddell & Kruschke, 2018). To this end, recent research in psychological methods has attempted to identify scenarios in which categorical variables can be treated as continuous in the context of Structural Equation Modeling and Network Psychometrics (see Johal & Rhemtulla, 2023; Rhemtulla et al., 2012).

In this paper, to be coherent with the ordinal nature of our data, we use entropy measures, including Shannon's conditional entropy and mutual information (please see the section "Entropy-based computations" for a description of these measures and for computational details). These measures are common in Information Theory and are used to evaluate associations between categorical variables (Furfaro & Hsieh, 2023). In addition, we coupled such measures with graphical representations and factor detection tools, which allow us to discover unique patterns in the variables while preserving the categorical nature of

the data. We developed code using the R programming language and using the libraries *gggraph* and *ggplot2* for generating the figures in this paper. The code is fully available at [blindforrevision](https://blindforrevision.com), making our analyses reproducible and allowing other researchers to apply similar techniques.

Before the data analyses, we removed all items and subjects that presented missing values in at least one of the items considered included in the questionnaire since they were a negligible group. The resulting sample size at each of the time points was: 213, 194, 209, and 210, respectively. For the longitudinal component of the analyses, we retained all participants with data at all four time points, resulting in a sample of 140 participants. The missing data regarded those participants who did not participate in all four waves. We decided not to impute missing data since this would have significant consequences on the heterogeneity of the features included in the dataset and we are indeed studying heterogeneity.

Items within a specific domain, i.e. within a specific Likert scale, clustered together and showed high associations (see Figures A1 and A2 in Appendix A where we used the same entropy measures to show association between items of the same domain). Therefore, we pooled all the items within each domain into a composite score for each measurement occasion using K-means (Hartigan & Wong, 1979). K-means is a popular clustering algorithm used in machine learning and data analysis which partitions a set of data points into K clusters. Each point is assigned to the cluster with the nearest mean, hence the name "k-means." We used K-means within each of the 11 Likert scales thus resulting in 11 new domain-specific variables, which we will simply refer to as *domain variables*. This method allowed to preserve the dependency among the items within a domain and to ensure even cluster sizes. For each of the 11 domain variables with four clusters containing 4 or 5 items for each time point, we encoded the final categorical responses as follows. First, we calculated the average across all items as the cluster-specific average. Next, we sorted out the orders of these four cluster-specific averages. By doing so, each domain-specific variable was transformed into a unidimensional ordinal variable. For instance, the domain variable "global self-worth" (*gsw*) had the following four ordinal categories:  $\{gsw = 1, gsw = 2, gsw = 3 \text{ and } gsw = 4\}$ , in increasing order of the clusters' averages of all items corresponding to the *gsw* domain. Thus, all new domain variables kept the original scale (1–4) to preserve the metric of the items and to facilitate the stability of domain-pairwise



Shannon conditional entropy calculations on a contingency table. Specifically, we made sure that a  $4 \times 4$  contingency table would have more than 13 averaged cell-counts. Because our longitudinal analyses also involved three dimensions simultaneously, we used a 3D categorical data cube. Thus, we expected entropy calculations to be based on a proportion of  $4^3$  of such 3-dimensional cubes due to the relations among the 11 domain variables.

### Entropy-based computations

To evaluate the associations between pairs of categorical variables, we used Shannon's conditional entropy and mutual information, two measures derived from Information Theory. In the next paragraphs, we define these measures and describe how we used them in our analyses.

Let  $Y$  be a categorical variable with values in  $S_Y$ , and  $X$  a categorical variable with values in  $S_X$ .  $Y$  acts here as a response variable, while  $X$  is considered the covariate. The associative relation between  $Y$  and  $X$  can be fully examined through their contingency table, denoted as  $C[X - vs - Y]$ , which is constructed by recording the number of subjects falling into the corresponding cell, with  $S_X$  and  $S_Y$  as the row- and column-axes, respectively. Following convention, we placed the response variable along the column-axis as outlined in the exemplified contingency table in Table 1. We will refer to this table as  $C[X - vs - Y]$ .

For each row of  $C[X - vs - Y]$ , conditional entropy is defined as Shannon's conditional entropy of  $Y$  given  $X$ . Let  $H(Y|X = x)$  be the entropy of variable  $Y$  conditioned on  $X = x$ . Hence, for the  $x$ -th row:

$$H[Y|X = x] = - \sum_{y \in S_Y} \frac{p(x, y)}{p(x)} \log \left( \frac{p(x, y)}{p(x)} \right). \quad (1)$$

We refer to  $H[Y|X = x]$  as *row-wise conditional entropy*. Here,  $p(x)$  denotes the proportion of  $x$ -th row-sum against the total sum, while  $p(x, y)$  denotes the proportion of  $(x, y)$  cell-count against the total sum. That is, the ratio  $\frac{p(x, y)}{p(x)}$  is used as the conditional probability  $p(Y = y|X = x)$ . We expect some levels of  $X$  to help reduce the entropy of our response variable and

**Table 1.** Example of  $4 \times 4$  contingency table with response categories on the columns.

| X | Y         |           |           |           |
|---|-----------|-----------|-----------|-----------|
|   | 1         | 2         | 3         | 4         |
| 1 | $n_{1,1}$ | $n_{1,2}$ | $n_{1,3}$ | $n_{1,4}$ |
| 2 | $n_{2,1}$ | $n_{2,2}$ | $n_{2,3}$ | $n_{2,4}$ |
| 3 | $n_{3,1}$ | $n_{3,2}$ | $n_{3,3}$ | $n_{3,4}$ |
| 4 | $n_{4,1}$ | $n_{4,2}$ | $n_{4,3}$ | $n_{4,4}$ |

therefore we will use this measure to see which levels of the covariates result in a greater reduction of entropy.

$H[Y|X = x]$  varies with respect to  $x$  across the row-axis of  $C[X - vs - Y]$ , therefore it is useful to summarize such variations with a (1) weighted average and (2) a measure of its variability across rows. More formally, we denote the weighted average of  $\{H[Y|X = x]|x \in S_X\}$  as:

$$H[Y|X] = - \sum_{x \in S_X} p(x) H[Y|X = x], \quad (2)$$

where  $H[Y|X]$  represents the Conditional Entropy (CE) of  $Y$  given  $X$ . The lower the conditional entropy, the more  $X$  contributes in explaining  $Y$ 's variation. When describing the association between variables related to global self-worth, we will look for those variables that contribute the most to a reduction in the entropy of global self-worth. In particular, we expect the variables that are theoretically considered as domains of global self-worth in Harter's model to show the lowest conditional entropy. The second quantity we utilize for summarizing  $H[Y|X = x]$  indicates the total-variation of the row-wise conditional entropy, which is denoted as follows:

$$\mathcal{V} < Y|X > = \sum_x H([Y|X = x] - H[Y|X])^2 \cdot p(x). \quad (3)$$

The value of  $\mathcal{V} < Y|X >$  is informative because it allows to quantify the amount of variability between the row-wise conditional entropies, thus permitting quantifying whether different levels of  $X$  contribute differently to lowering the entropy of  $Y$ .

With regards to the directional association from  $X$  to  $Y$ , in this paper, we measure it using the re-scaled conditional entropy:

$$CE^{res}[Y|X] = \frac{H[Y|X]}{H[Y]}. \quad (4)$$

Here,  $CE^{res}[Y|X]$  is the remaining proportion of uncertainty of  $Y$  given the knowledge of  $X$ , which makes this re-scaled entropy ratio a reasonable measurement of directional association between  $X$  and  $Y$ .

Moreover, we define the mutual information of  $Y$  and  $X$  as:

$$I[Y : X] = H[Y] - H[Y|X] = H[X] - H[X|Y].$$

This  $I[Y : X]$  will be used to identify which variables contribute more to reduce uncertainty in  $Y$ .

Finally, we define mutual conditional entropy (MCE) as the following average:

$$MCE[X; Y] = \left\{ \frac{H[Y|X]}{H[Y]} + \frac{H[X|Y]}{H[X]} \right\} / 2. \quad (5)$$

This  $MCE[X; Y]$  is used as an informative measurement for non-directional associations between two variables.  $I[Y : X]$  is not as informative as  $MCE[X; Y]$  for evaluating associations because the marginal entropies  $H[Y]$  and  $H[X]$  could be significantly different. We use this measure to assess pair-wise associations between variables, and we use networks with domains as nodes and MCE as links and heatmaps to represent the magnitude of the pairwise MCEs.

Conditional Entropy, as given in Equation (4), has been used to develop an approach to identify the most important variables that marginally contribute to explaining the entropy in an outcome. This recently developed approach is called “major factor selection” (Chen et al., 2021; Chou et al., 2022; Hsieh et al., 2022). To identify major factors beyond the marginal association, we use a data-splitting approach (Hsieh et al., 2022). This technique consists of splitting the dataset based on the levels of a major factor and running the major factor analysis on each subset of the data. Since this approach identifies major factors beyond the first one, the data splitting approach can be seen as a technique to identify second-order major factors. In our analyses, we use major factor selection and data splitting to identify main collections of variables, called major factors, of various orders underlying the dynamics of the outcome variable and its covariates.

## Results

### Associations and connectivity

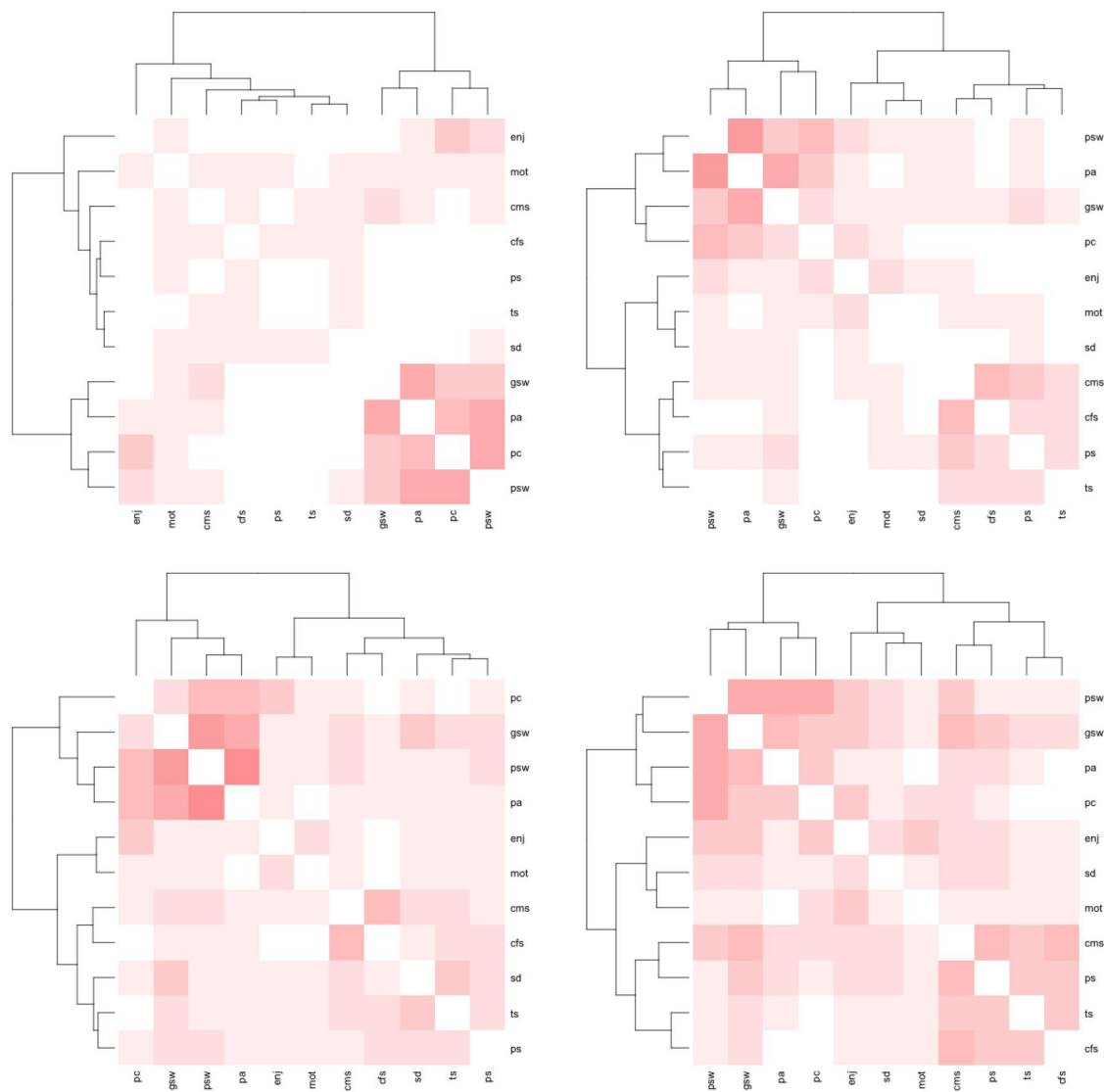
We first examined the pair-wise associations among the domain variables at each time point. We conducted these analyses separately for each measurement occasion. We plotted each of the  $11 \times 11$  symmetric MCE matrices as heatmaps, one per measurement occasion. The four heatmaps are represented in the four panels of Figure 1, and they show varying degrees of block-pattern associations. We used an MCE threshold of 0.8 to visualize the observed values of MCE using the network representation, and these are displayed in Figure 2. Both Figures 1 and 2 point to patterns of non-stationarity across the four time points, as the strength of the associations changes over time. In particular, while at the beginning of the semester global self-worth seemed solely associated with perceived appearance and competence and physical self-worth, as students move on through the semester, it seems that global self-worth is more associated with the importance of others’ support (close friends, parents, etc.).

### Directional associations among domain variables

To examine directional associations among the 11 domain variables, we used the rescaled conditional entropy  $CE^{res}$  in Equation (4) and we represented it using directed networks with the domain variables as nodes. These networks, one for each time point, are depicted in the four panels of Figure 3. Through these four panels, we observed that global self-worth (*gsw*) is indeed linked to many other variables, and this is true across all four time points, suggesting that global self-worth is the variable that best summarizes the other constructs considered. This finding validates our decision of considering *gsw* as the focus or outcome variable in further analyses, and all the other variables as explicatory variables, i.e. covariates. In addition, this finding suggests that directional networks based on  $CE^{res}[X_q|X_s]$  can be used as an alternative to path analysis, in which linear modeling structures have to be assumed.

At each measurement occasion, we examined the conditional entropy of *gsw* given the four possible ordinal values of the covariates. For instance, taking *pa* (physical appearance) as the covariate, we calculated  $H[gsw|pa = 1]$ ,  $H[gsw|pa = 2]$ ,  $H[gsw|pa = 3]$ , and  $H[gsw|pa = 4]$  based on the contingency table  $C[pa - vs - gsw]$  for the corresponding time point. Looking at Figure 4, these four values are displayed in the top left group of panels, in the first sub-panel of the second row, for the first time stamp (light blue). As displayed in the figure, the entropy of *gsw* is higher for lower values of physical appearance and decreases as this covariate increases. Starting from the contingency table defined by  $C[cfs - vs - gsw]$ , each row of  $C[cfs - vs - gsw]$  defines a multinomial distribution with parameters given by the corresponding row-sum and row-proportion vector. Based on these values, we simulated 1,000 contingency tables and we computed 1,000 conditional entropy values, thus simulating the distribution of the conditional entropy from which a confidence bar was computed. These confidence intervals are represented in Figure 4 as the bars that extend above and below the observed values of conditional entropy.

From the 10 sub-panels in Figure 4, we observed that there is an apparent decreasing pattern along the ordinal axis from values 1 to 4. These almost linear patterns are crucial for understanding the relation between global self-worth and the covariates. Focusing for example on physical appearance (panel titled *pa*), we can see that as adolescents feel more confident about their physical appearance in levels 3 and 4, the entropy of global self-worth is the lowest, meaning



**Figure 1.** Heatmaps of MCE for the 11 domain variables. From top left corner, clockwise: Time-1, Time-2, Time-3, Time-4.

that adolescents in high levels of perceived physical appearance tend to report similar levels of global self-worth.

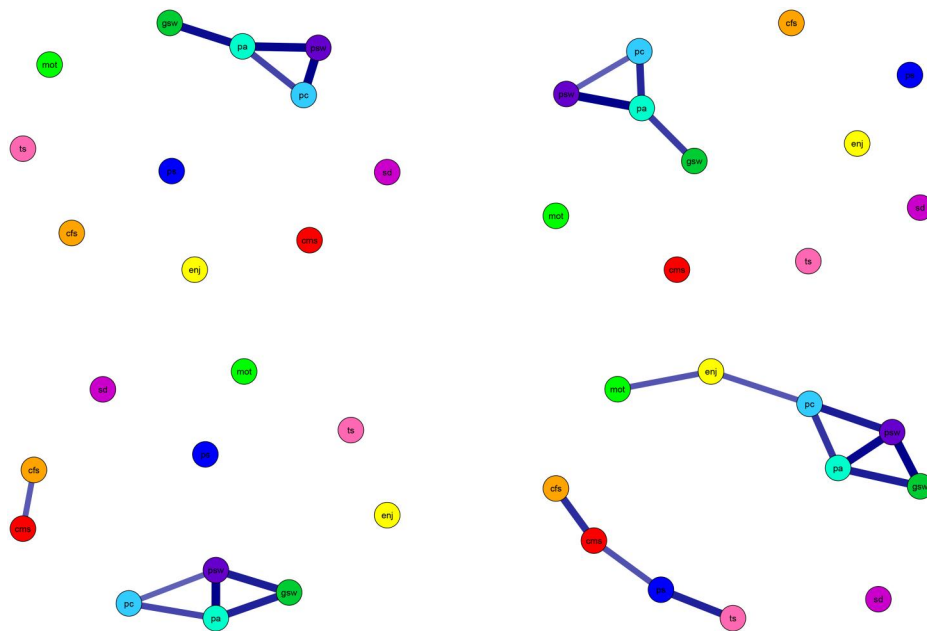
The slope of the almost linear decline is captured by  $\mathcal{V} < gsw | X_s >$ , the total variation between the four conditional entropy values. The ordinal nature of the ten domain variables denoted as  $X_s$ , brings about visible patterns of information by increasing the certainty in predicting  $gsw$ . Thus, the two emerging patterns of information can be described by: (a) the explained uncertainty *via*  $1 - CE^{res}[gsw | X_s]$ , and (b) the nearly linear decline *via*  $\mathcal{V} < gsw | X_s >$ . We report these two aspects in Table 2.

From Table 2, we observed that the covariates that explain the most uncertainty in  $gsw$  are physical appearance and physical self-worth (at time-1 and time-2), and physical self-worth and physical

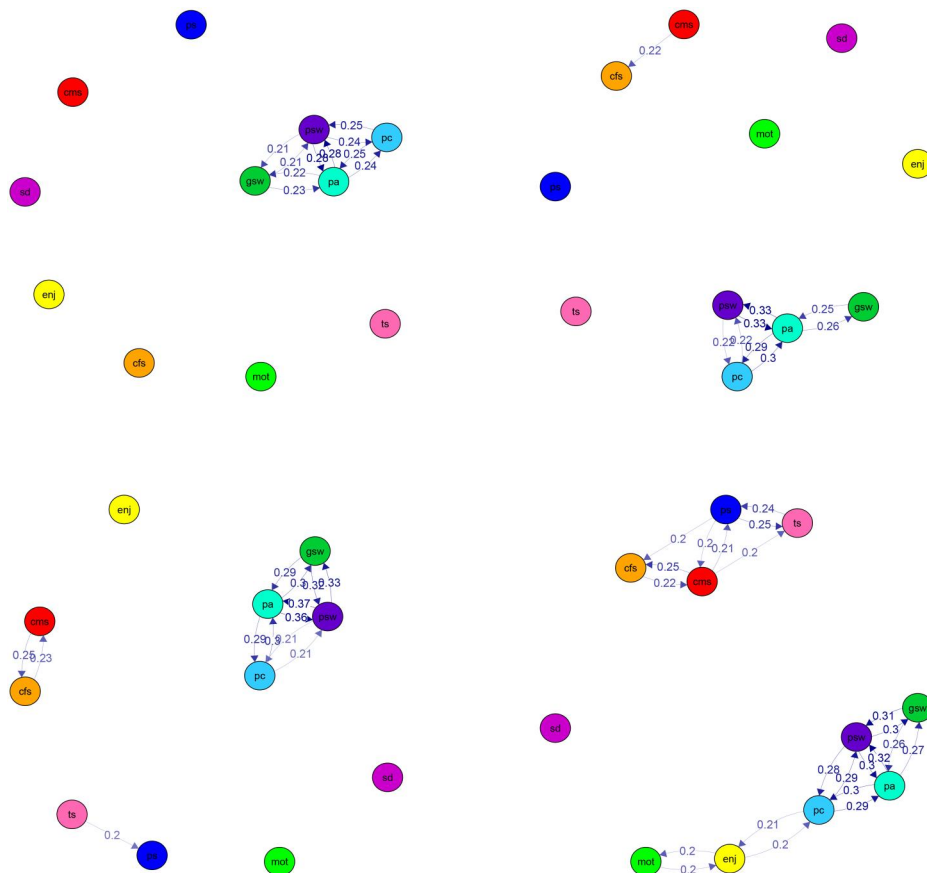
appearance (at time-3 and time-4). This indicates that most of the variability observed in global self-worth is linked in some way to the way these adolescent students perceive their “physical self.” The 3rd and 4th ranked variables vary greatly across all four time points. These findings reinforce once again the non-stationarity in the structure of the data across time.

### **Temporal evolution of the structure of self-worth**

Next, we examined changes in the structure of the self-worth model over time. Given the non-stationarity detected in the previous sections, the dynamics underlying global self-worth and its entire network are unlikely to be governed by a single mechanism. We used the major factor identification (see Hsieh et al., 2022) to get each temporal-phase of the



**Figure 2.** MCE represented as a network. From top left corner, clockwise: Time-1, Time-2, Time-3, Time-4. A linkage between nodes is presented if  $1 - MCE > 0.2$ .

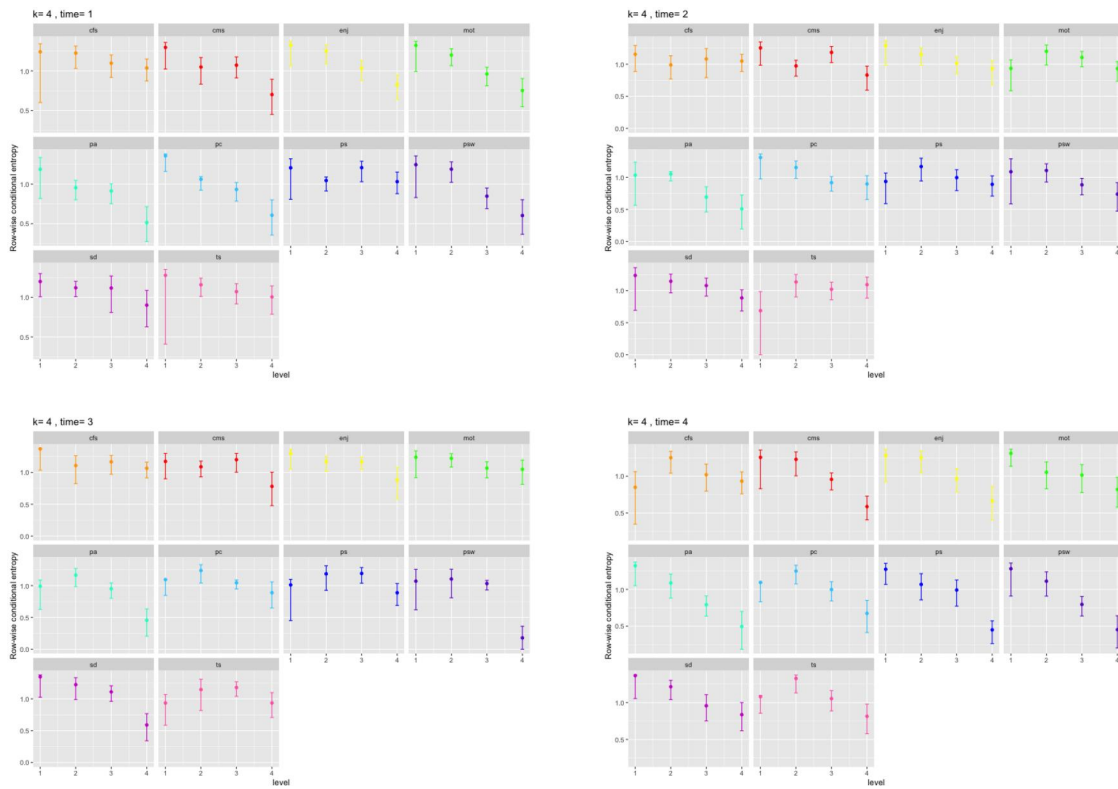


**Figure 3.** Rescaled conditional entropy represented using networks of directional associations among the 11 domain variables. A linkage between nodes is represented if  $1 - CE^{res}[X_i|X_s] > 0.2$ . From top left corner, clockwise: Time-1, Time-2, Time-3, Time-4.

longitudinal analysis and map out the phase-specific heterogeneity. The four phases of our longitudinal analysis, aided by major factor selection protocol, are

described with respect to the four time points. To clarify the time point for each domain variable, we use the suffix “1,” “2,” “3,” and “4” to denote time.





**Figure 4.** Row-wise conditional entropy of  $gsw$  given levels of each covariate  $H(gsw|X_s = x_s)$ . From top left corner, clock-wise: Time-1, Time-2, Time-3, Time-4. Confidence bars are calculated via row-wise simulated multinomial distribution.

**Table 2.** Uncertainty (and variation) of  $gsw$  explained by the different variables:  $1 - CE^{res}[gsw|X_s]$  and in parenthesis  $\mathcal{V} < gsw|X_s >$  for each time point.

| Variables | Time-1        | Time-2        | Time-3        | Time-4        |
|-----------|---------------|---------------|---------------|---------------|
| pc        | 0.190 (0.056) | 0.137 (0.020) | 0.180 (0.016) | 0.182 (0.043) |
| pa        | 0.303 (0.048) | 0.313 (0.051) | 0.343 (0.077) | 0.300 (0.075) |
| psw       | 0.249 (0.058) | 0.212 (0.020) | 0.410 (0.151) | 0.344 (0.077) |
| ps        | 0.043 (0.007) | 0.169 (0.009) | 0.159 (0.021) | 0.234 (0.089) |
| ts        | 0.047 (0.005) | 0.102 (0.011) | 0.154 (0.013) | 0.133 (0.031) |
| cfs       | 0.034 (0.006) | 0.088 (0.003) | 0.106 (0.007) | 0.152 (0.015) |
| cms       | 0.143 (0.037) | 0.095 (0.024) | 0.162 (0.025) | 0.256 (0.066) |
| mot       | 0.127 (0.040) | 0.070 (0.011) | 0.100 (0.007) | 0.117 (0.027) |
| enj       | 0.065 (0.031) | 0.074 (0.012) | 0.108 (0.018) | 0.194 (0.053) |
| sd        | 0.054 (0.010) | 0.089 (0.013) | 0.210 (0.073) | 0.136 (0.035) |

The first phase of the longitudinal analysis considers time-1. It begins by employing  $gsw1$  (i.e., global self-worth at time 1) as the focus variable and the remaining ten domain variables as covariates. However, for the sake of brevity, we extensively present results only for times 2 and 4, while results for timepoints 1 and 3 are available in [Appendix A](#) (for time 1, see [PICK Tables A2–A4](#); for time 3, see [Figures A3 and A4, Tables A5–A7](#)).

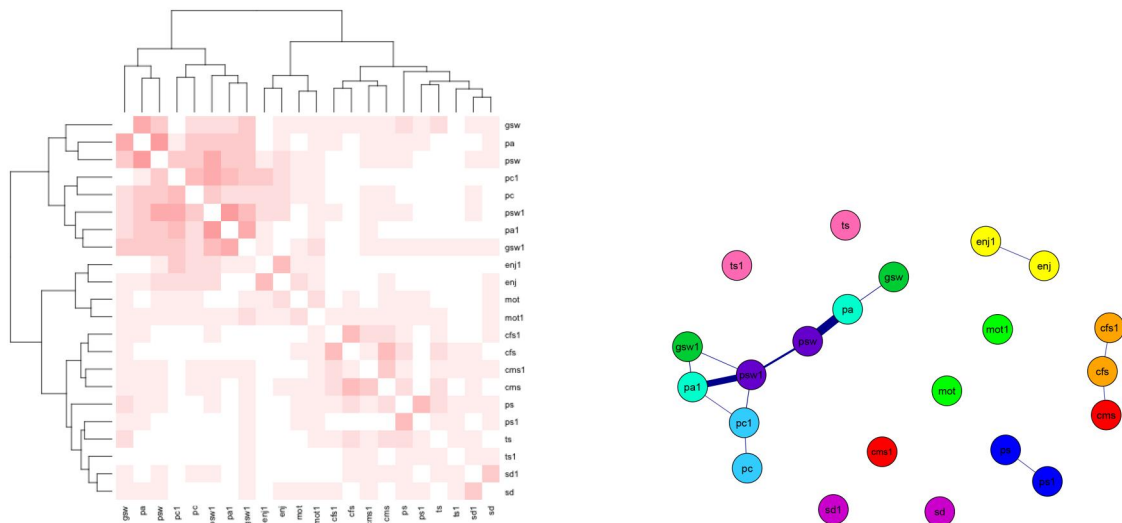
### Including longitudinal data in the sequence

At time-2, the response variable is global self-worth at time-2 ( $gsw2$ ) and the covariates include the 10 domain variables at time-2 as well as those at time-1,

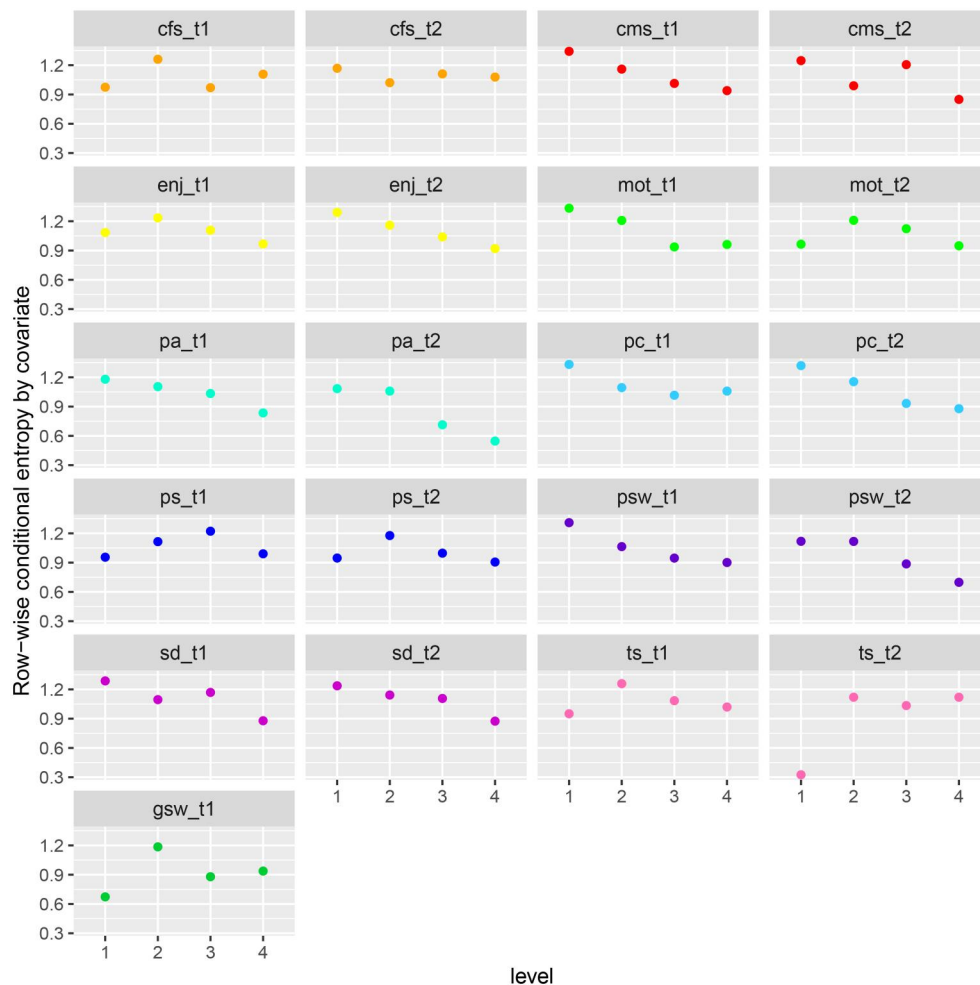
in addition to global self-worth at time-1 ( $gsw1$ ). The  $22 \times 22$  MCE heatmap and corresponding network with the same thresholding as before ( $1 - MCE > 0.2$ ) are presented in the two panels of [Figure 5](#).

Observing [Figure 5](#), we can see that  $cfs1$ ,  $cfs2$ , and  $cms2$  are connected in the network. This means that the perception of close friends support at time 2 depends on that at time 1 and it is also associated with class-mates support, suggesting that classmates and close friendships are becoming interconnected. Upon a finer scale view, conditional entropies with respect to the four ordinal categories of all 21 covariates are displayed in [Figure 6](#). Many covariates show decreasing or even nearly linear declining patterns, such as  $cms1$ ,  $enj2$ ,  $mot1$ ,  $pa1$ ,  $pa2$ ,  $pc1$ ,  $pc2$ ,  $psw1$ ,  $psw2$ ,  $sd1$ , and  $sd2$ . In particular,  $pa2$  has a steeper decline than that of  $pa1$ , so do  $psw2$  against  $psw1$ , and  $pc2$  against  $pc1$ . Though these steep declining patterns are intuitive, they are strong indicators of heterogeneity along the ordinal categorical axes of almost all covariates.

As part of the major factor selection, in [Table 3](#) we show the ranked conditional entropies of  $gsw2$  given all 21 covariates at time-2. One noticeable result is that global self-worth at time-1 ( $gsw1$ ) is ranked 3rd behind physical appearance at time-2 ( $pa2$ ) and physical self-worth at time-2 ( $psw2$ ). This order indicates



**Figure 5.** Heatmap of  $22 \times 22$  matrix of MCE pertaining to 22 domain-specific variables at time-2 (Left panel). Network of associations between domain variables (Right panel). A linkage between nodes is presented if  $1 - \text{MCE} > 0.2$ .



**Figure 6.** Row-wise conditional entropies of  $gsw2$  given levels of each covariate  $H(gsw2|X_s = x_s)$ . The suffix “t” is added here to highlight the time point the variable refers to.

that the physical characteristics perceived currently are more important than the perceived global self-worth as reported at the previous time point. The

results reported in Table 3 support the patterns displayed in Figure 6. To identify covariates that can provide extra information beyond the four ranked at

the top, we consider the data splitting technique used by Hsieh et al. (2022). We split the entire sample at time-2 into three subgroups by conditioning on the four ordinal categories of each of the top ranked covariates. The sample sizes for the resulting subgroups are reported in Table 4.

Table 5 reports the sets of three variables with the highest association for each conditional covariate based on conditional entropy. This table shows differences in the triplets for each of the conditioning variables and across the three subgroups. For instance, when considering *pa2* as the conditioning covariate, *gsw1* appears as important for the first two subgroups, but not the third. This specific aspect of heterogeneity indicates that *gsw1* can provide extra information beyond *pa2* only locally, not for the entire sample. Similar patterns of heterogeneity are also evident in the next two time points.

When including data for the third measurement occasion, the response variable was *gsw3* and there were 32 covariates, including *gsw1* and *gsw2*. The analyses were carried out in the same fashion as described for time-2. For the sake of brevity, we only present these in Appendix A in Tables A5–A7.

**Table 3.** Uncertainty and variation of *gsw2* explained by the different variables:  $1 - CE^{res}(gsw2|X_s)$  and  $\mathcal{V} < gsw2|X_s >$  for time-2.

|    | Covariate | $1 - CE^{res}(gsw2 X_s)$ | $\mathcal{V} < gsw2 X_s >$ |
|----|-----------|--------------------------|----------------------------|
| 1  | pa2       | 0.300                    | 0.048                      |
| 2  | psw2      | 0.223                    | 0.026                      |
| 3  | gsw1      | 0.213                    | 0.012                      |
| 4  | ps2       | 0.174                    | 0.008                      |
| 5  | psw1      | 0.162                    | 0.013                      |
| 6  | pc2       | 0.144                    | 0.021                      |
| 7  | pa1       | 0.140                    | 0.014                      |
| 8  | ts2       | 0.120                    | 0.032                      |
| 9  | mot1      | 0.101                    | 0.020                      |
| 10 | cms1      | 0.098                    | 0.015                      |
| 11 | sd2       | 0.093                    | 0.015                      |
| 12 | cms2      | 0.090                    | 0.023                      |
| 13 | ps1       | 0.080                    | 0.010                      |
| 14 | enj2      | 0.074                    | 0.013                      |
| 15 | cfs2      | 0.070                    | 0.003                      |
| 16 | sd1       | 0.067                    | 0.016                      |
| 17 | cfs1      | 0.067                    | 0.011                      |
| 18 | mot2      | 0.066                    | 0.010                      |
| 19 | pc1       | 0.064                    | 0.011                      |
| 20 | enj1      | 0.053                    | 0.009                      |
| 21 | ts1       | 0.049                    | 0.009                      |

### Complete longitudinal sequence

At the final time point, the response variable was *gsw4* and the analysis included all other variables at previous time points together with global self-worth at previous times, thus totaling 43 covariates. We started these analyses by examining the association patterns among these 44 variables. These associations are presented in Figure 7. The heatmap in this figure includes more blocks of associations and the network more linkages of connectivity with higher degrees of complexity than those observed at previous time points, including Figure 5. Several isolated nodes in previous networks (see Figure 5) now show connections in Figure 7. For example, the variables enjoyment (*enj*), self-determination (*sd*), and class-mate support (*cms*) now show higher degrees of connection in the network.

The conditional entropies of *gsw4* as a function of the ordinal categories of the covariates are displayed in Figure 8. The successive panels across each time point reveal the evolving patterns within each domain. In particular, the variables *enj*, *pa*, *psw*, and *sd* show a steep decline at all time points, and the pattern becomes stronger across time. This pattern indicates that the uncertainty in *gsw4* decreases with the increasing ordinal value of the responses of the covariates. This trend becomes stronger across time as, for instance, *psw* at time four shows a steeper decline than *psw* at time one. This result indicates that for those students reporting high values in physical self-worth, global self-worth becomes more stable over time. Such fine-scale information is fundamental to understand the evolution of the structure of the self in our data.

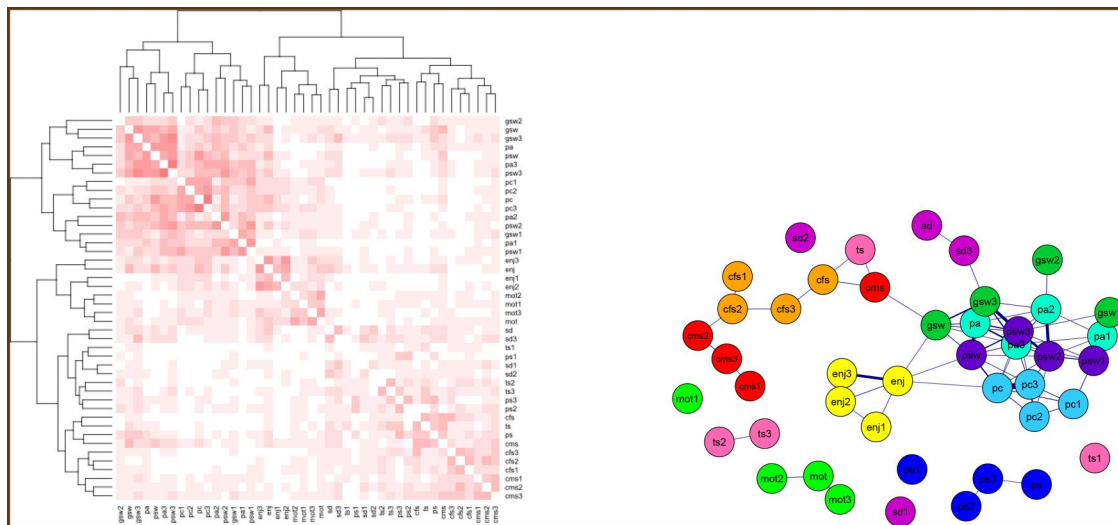
Next, we repeated the computation of the weighted averages and the total variations of the four ordinal

**Table 5.** Triplets providing the most information (highest  $CE^{res}$ ) for each conditioning variable across each category at time 2.

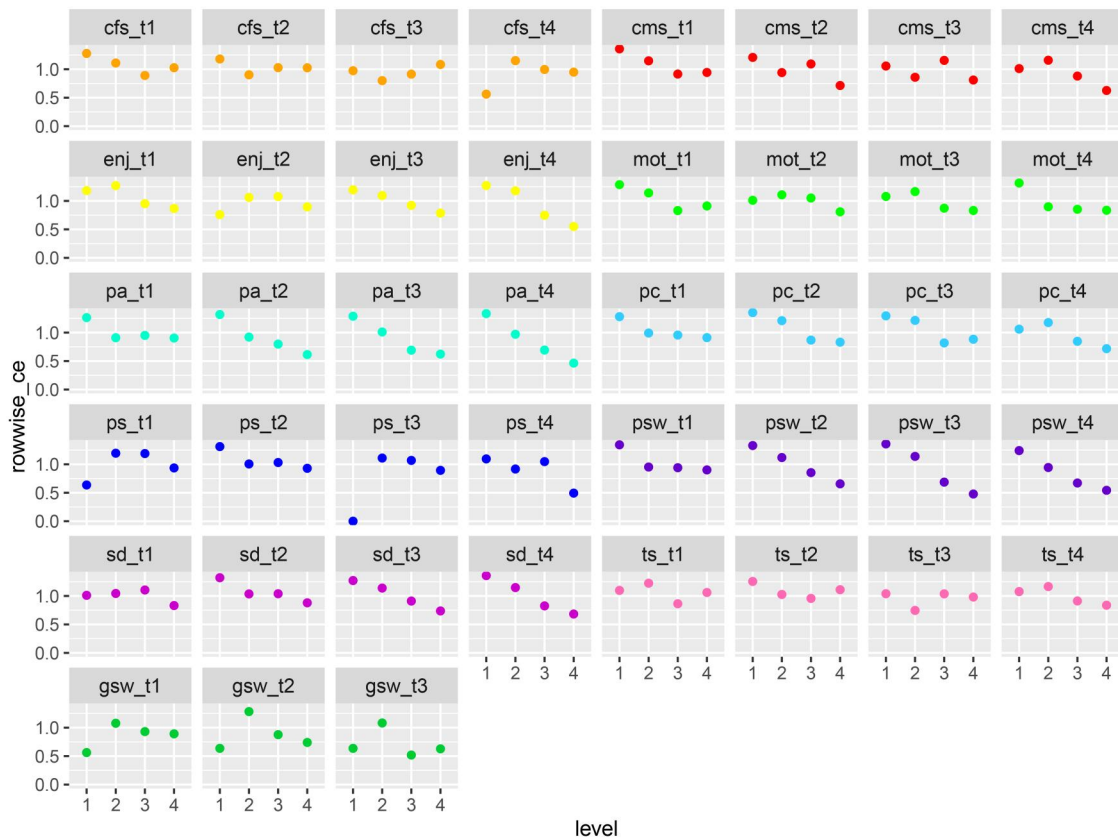
| Conditioning variable | Category          |                  |                   |
|-----------------------|-------------------|------------------|-------------------|
|                       | 1 OR 2            | 3                | 4                 |
| pa2                   | (gsw1, ps2, psw1) | (gsw1, ts2, ps1) | (enj1, mot2, pc1) |
| psw2                  | (pa2, psw1, gsw1) | (ts2, ps1, ps2)  | (pa1, cfs2, pa2)  |
| gsw1                  | (pa2, cfs2, enj1) | (pa2, ps2, pc2)  | (ts2, cfs2, cms2) |
| ps2                   | (psw2, pa2, pc2)  | (pa2, pa1, gsw1) | (gsw1, pa1, enj1) |

**Table 4.** Number of subjects per response category (columns) across the four major covariates at time 2 (rows).

| Conditioning variable | Category = 1 | Category = 2 | Category = 3 | Category = 4 |
|-----------------------|--------------|--------------|--------------|--------------|
| pa2                   | 22           | 66           | 50           | 37           |
| psw2                  | 19           | 46           | 72           | 38           |
| gsw1                  | 5            | 25           | 70           | 75           |
| ps2                   | 16           | 26           | 59           | 74           |



**Figure 7.** Heatmap of  $44 \times 44$  matrix of MCE pertaining to 44 domain variables at time-4 (Left panel). Network of associations among domain variables (Right panel). A linkage between nodes is presented with a threshold of  $1 - \text{MCE} > 0.2$ .



**Figure 8.** Row-wise conditional entropies of  $gsw2$  given levels of each covariate  $H(gsw4|X_s = x_s)$ . The suffix “t” is added here to highlight the time point the variable refers to.

categories for all 43 covariates. These results are reported in Table 6. Global self-worth at time 3  $gsw3$  shows the largest reduction in uncertainty in  $gsw4$  as well as the largest total variation. This is surprising because global self-worth did not show similar results

at time 3 or time 2. In contrast,  $psw4$  and  $pa4$  emerge as the next two covariates with the largest reduction in uncertainty in  $gsw4$ , in line with findings from previous time points supporting the importance of these two domains.

To investigate which variables are candidates for order-1 major factors beyond other variables, we considered the top four ranked covariates as candidates.

**Table 6.** Uncertainty and variation of  $gsw4$  explained by the different variables:  $1 - CE^{res}(gsw4|X_s)$  and  $\mathcal{V} < gsw4|X_s >$  for time-4.

|    | Covariate | $1 - CE^{res}(gsw4 X_s)$ | $\mathcal{V} < gsw4 X_s >$ |
|----|-----------|--------------------------|----------------------------|
| 1  | gsw3      | 0.377                    | 0.048                      |
| 2  | psw4      | 0.349                    | 0.043                      |
| 3  | pa4       | 0.316                    | 0.069                      |
| 4  | psw3      | 0.314                    | 0.080                      |
| 5  | pa3       | 0.270                    | 0.045                      |
| 6  | enj4      | 0.259                    | 0.070                      |
| 7  | cms4      | 0.253                    | 0.037                      |
| 8  | ps4       | 0.234                    | 0.056                      |
| 9  | pa2       | 0.221                    | 0.041                      |
| 10 | pc4       | 0.203                    | 0.026                      |
| 11 | gsw2      | 0.188                    | 0.037                      |
| 12 | sd4       | 0.172                    | 0.052                      |
| 13 | psw2      | 0.165                    | 0.042                      |
| 14 | gsw1      | 0.164                    | 0.007                      |
| 15 | ts4       | 0.147                    | 0.013                      |
| 16 | mot4      | 0.145                    | 0.029                      |
| 17 | enj3      | 0.135                    | 0.017                      |
| 18 | pa1       | 0.133                    | 0.009                      |
| 19 | pc3       | 0.131                    | 0.032                      |
| 20 | sd3       | 0.129                    | 0.031                      |
| 21 | cms3      | 0.126                    | 0.021                      |
| 22 | psw1      | 0.120                    | 0.013                      |
| 23 | cms2      | 0.118                    | 0.028                      |
| 24 | mot3      | 0.116                    | 0.022                      |
| 25 | mot1      | 0.111                    | 0.023                      |
| 26 | pc2       | 0.107                    | 0.035                      |
| 27 | ts3       | 0.106                    | 0.009                      |
| 28 | cfs4      | 0.099                    | 0.010                      |
| 29 | cfs3      | 0.094                    | 0.011                      |
| 30 | mot2      | 0.088                    | 0.015                      |
| 31 | ps3       | 0.087                    | 0.016                      |
| 32 | pc1       | 0.087                    | 0.013                      |
| 33 | ps2       | 0.081                    | 0.009                      |
| 34 | sd1       | 0.081                    | 0.010                      |
| 35 | enj2      | 0.076                    | 0.011                      |
| 36 | cms1      | 0.075                    | 0.019                      |
| 37 | sd2       | 0.074                    | 0.011                      |
| 38 | cfs1      | 0.072                    | 0.010                      |
| 39 | ts1       | 0.067                    | 0.021                      |
| 40 | cfs2      | 0.059                    | 0.009                      |
| 41 | enj1      | 0.055                    | 0.025                      |
| 42 | ts2       | 0.048                    | 0.008                      |
| 43 | ps1       | 0.031                    | 0.020                      |

As was the case at previous time points, we subdivided the sample into subgroups based on the four ordinal categories of each variable. The resulting sample sizes of these subgroups are reported in Table 7. Due to the small size of the first group, we again collapsed the first two subgroups, making our procedure consistent across all four time points.

Table 8 contains the three-variable triplets with the highest associations for each of the top ranked covariates across the three subgroups. Some results are noticeable from this table. First, when conditioning on  $gsw3$ , the three top triplets presented in the 1st row are all covariates at time-4, indicating that, at the last measurement occasion, global self-worth is associated with concurrent domains, in addition to self-worth at the previous occasion. Second, the covariates providing the most information beyond  $gsw3$  are different for the three subgroups. For example,  $psw4$  is the most important for subgroup *no.1&2*,  $pa4$  is the most important for subgroup *no.3*, and  $cms4$  for subgroup *no.4*. When conditioning on  $psw4$ ,  $gsw3$  is the variable providing the highest amount of information for subgroups *no.1&2* and *no.3*, but not for subgroup *no.4*. For this subgroup, the past values of physical self-worth ( $psw3$ ) and enjoyment ( $enj3$ ) provide the largest extra information beyond  $psw4$ . This heterogeneity is also found when conditioning on  $pa4$  or  $ps3$ . Third,  $gsw3$  appears in most triplets for subgroups *no.1&2* and *no.3*, but not for subgroup *no.4*. As was the case in previous analytic steps, this result points to the heterogeneity of the data and, more precisely, to the heterogeneity in the underlying dynamics of self-worth.

## Discussion

In this study, we examined Harter's (Harter, 1988, 1999) model of self-worth together with some potential

**Table 7.** Number of subjects per response category (columns) across the four major covariates at time 4 (rows).

| Conditioning variable | Category = 1 | Category = 2 | Category = 3 | Category = 4 |
|-----------------------|--------------|--------------|--------------|--------------|
| gsw3                  | 6            | 34           | 42           | 58           |
| psw4                  | 11           | 30           | 55           | 44           |
| pa4                   | 14           | 35           | 54           | 37           |
| psw3                  | 12           | 25           | 65           | 38           |

**Table 8.** Triplets providing the most information (highest  $CE^{res}$ ) for each conditioning variable across each category at time 4.

| Conditioning variable | Category           |                   |                    |
|-----------------------|--------------------|-------------------|--------------------|
|                       | 1 OR 2             | 3                 | 4                  |
| gsw3                  | (psw4, pa4, ts4)   | (pa4, sd4, enj4)  | (cms4, enj4, psw4) |
| psw4                  | (gsw3, ts4, enj2)  | (gsw3, enj3, ps4) | (psw3, enj3, enj4) |
| pa4                   | (gsw3, mot4, gsw2) | (enj4, gsw3, ps4) | (enj4, cms4, ps4)  |
| ps3                   | (gsw3, cms4, pa3)  | (ps4, enj4, pa2)  | (enj4, psw4, cms4) |



antecedents and consequences for a sample of adolescent students in high school. We evaluated the structure of the association between different dimensions of self-worth together with its changes over time during the course of a semester in high school. For this, we represented the model of self-worth as a network and implemented a data analytics technique based on mutual conditional entropy. Using this technique, we investigated changes in the structure of the model of self-worth across time, as well as associations between the various dimensions of the model in such changes.

### Summary of findings

Several findings from our analyses are worth mentioning. First, the different dimensions integrating self-worth showed interconnections at each measurement occasion. This was expected based on the theoretical foundation of the model (Harter, 1999). Furthermore, such associations became stronger across time. That is, our measures of mutual entropy showed that the dimensions comprising self-worth showed stronger interconnections as time passed by. This finding could be due to the context specific nature of self-worth and the fact that these data were collected during the students' first semester in high school. In this situation, it takes time for the various self-evaluations to develop and for its various components to exert their expected influences on the self.

A second important finding concerns the analyses examining the time evolution of the model. We found several groups of individuals who showed different patterns of associations among the various components of the self-worth model. Some key aspects of such differences involved "perceived appearance," "global self-worth," and "physical self-worth." Similar patterns have been consistently found in the literature (Erdvik et al., 2020). For example, Haugen et al. (2011) found that perceived physical appearance was related to global self-worth among adolescents by serving as a mediator from physical activity and increasing their perceptions of physical self-esteem. This influence of physical appearance on global self-worth appeared to be stronger for females than for males. Furthermore, our analyses indicate that the way these dimensions influence each other over time differs across subgroups of individuals, highlighting the heterogeneity in the data. Third, we found that "global self-worth" truly emerges as an important factor in the model at the third measurement occasion and that, over time, it becomes more strongly associated with concurrent

variables rather than with variables at preceding occasions.

### Theoretical and methodological considerations

Our data consisted of self-reports taken on four measurement occasions. For the present analyses, we included 50 items tapping onto 11 dimensions of self-worth, each dimension measured using a Likert scale. When completing the questionnaires, students were asked to rate each item on a scale of 1–4. These responses represent anchor points of an underlying construct and, while they preserve the order i.e.,  $4 > 3 > 2 > 1$ , the distance between categories cannot be assumed to be equal. Although treating data like these as continuous is a common practice, procedures for continuous data may produce misleading results (Agresti, 2003; Goodman, 1978; Liddell & Kruschke, 2018). The motivation underlying our proposed computational approach based on entropy measures resides in the fact that we want to extract information without assuming or imposing distances between the responses. The original ordinal scale:  $\{1, 2, 3, 4\}$ , under the common practice of numerical encoding scheme could be replaced by  $\{1, 5, 7, 13\}$ , or even by  $\{a, b, c, d\}$ . It is important to keep in mind that the distance between code-1 and code-2 is not equal to that between code-3 and code-4, and so on. Moreover, every respondent potentially implements a different scheme to encode his or her response to the various items. Thus, it is not reasonable to presume that there is a single common measurement scale across questions or across individuals. Without such a unified scale underlying our self-report data, the statistical concepts of mean, variance, and correlation as in continuous variables are difficult to justify.

Linear models are the standard approach to study associations and dependencies between variables, either categorical or continuous, assuming linear relationships. Linear models are intrinsically centered around means, variance, and correlations. Moreover, correlations are always symmetric, i.e. the linear trend of two variables is the same no matter which variable is placed on which axis. Conditional entropy, on the other hand, is not. Computations based on entropy, as those used extensively in this paper, are intrinsically categorical and the computed directional associations can be asymmetric. Since the nature of categorical data is embraced by all types of data, our proposed approach provides a more general framework in which both categorical and continuous variables can be implemented. Future work should examine potential differences in

the results obtained when using our proposed approach vs. techniques based on linear models. We expect linear models to provide results in line with ours when the associations are linear. However, when the associations are not linear, we expect our method to be able to discern patterns of associations between variables that linear models cannot detect.

To visualize the associations between variables and their various response categories, we used a network representation. In this regard, our approach can be related to network psychometrics (Borsboom et al., 2021; Epskamp, 2020; Isvoranu et al., 2022; Steinley, 2021). In a psychometric network, the focus is primarily placed on the patterns of pairwise associations conditioning on the rest of variables, usually through partial correlations and under linearity assumptions (Borsboom et al., 2021; Epskamp, 2020). In contrast, the goal of our approach is to identify the dynamics of our selected outcome, general self-worth, as a function of the other ten variables in the data. We expect such dynamics of self-worth to be related to several variables as well as interacting effects of various orders, across the four measurement occasions. In our analyses, all computed directional associations with global self-worth can be taken as directed relational connections and are evaluated using measures of conditional entropy. Similarly, if the outcome measure is replaced by any other variable in the data, the same computations would weave a directional network among the eleven dimensions. To be clear, our goal with this paper is not to compare our proposed approach with network psychometrics, nor we claim that our methodology is superior in any way. We are simply implementing a method that we believe can capture the intended target question given the characteristics of our data.

In future work, we intend to make use of a single-feature based computations to offer an alternative way of building a network with directional linkages based on all 11 possible combinations of the ten covariates directed toward the designated response of the outcome. These directed linkages would be either weighted or binary with respect to a threshold. The directed network can be represented by an 11-by-11 covariate by response network, with the response categories of the covariates arranged along the row axis and the outcome response categories along the column-axis, and with each entry being the weight or coded as 0 or 1, to denote a directed linkage.

Despite the described potential advantages of our proposed approach, one important limitation is that it cannot provide straightforward information, such as

that yielded by linear models. One aspect of our approach that needs further examination is about sampling variability. In this paper, we took sampling variability into account by computing confidence bands for measures of conditional entropy based on a multinomial distribution with probabilities estimated using the sample data. Future research should focus on further quantifying sampling variability and on the detection of highly unpredictable subjects. Identifying such outliers would help understand who contributes to the estimation of sampling variability.

## Conclusion

To conclude, from a theoretical standpoint our findings provide support to Harter's model of self-worth (Harter, 1988, 1999). The interconnections among the different dimensions of the model at each occasion together with the increased strength of such associations over time support this network of variables as a representation of the self. Methodologically, our approach emphasizes the importance of treating categorical data as such and of identifying patterns of heterogeneity. Furthermore, our longitudinal network analyses highlight the need of examining the evolution of the network to identify its dynamics. We hope our approach stimulates more studies that focus on the dynamics of multivariate systems.

## Article information

**Conflict of interest disclosures:** Each author signed a form for disclosure of potential conflicts of interest. No authors reported any financial or other conflicts of interest in relation to the work described.

**Ethical principles:** The authors affirm having followed professional ethical guidelines in preparing this work. These guidelines include obtaining informed consent from human participants, maintaining ethical treatment and respect for the rights of human or animal participants, and ensuring the privacy of participants and their data, such as ensuring that individual participants cannot be identified in reported results or from publicly available original or archival data.

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## Appendix A

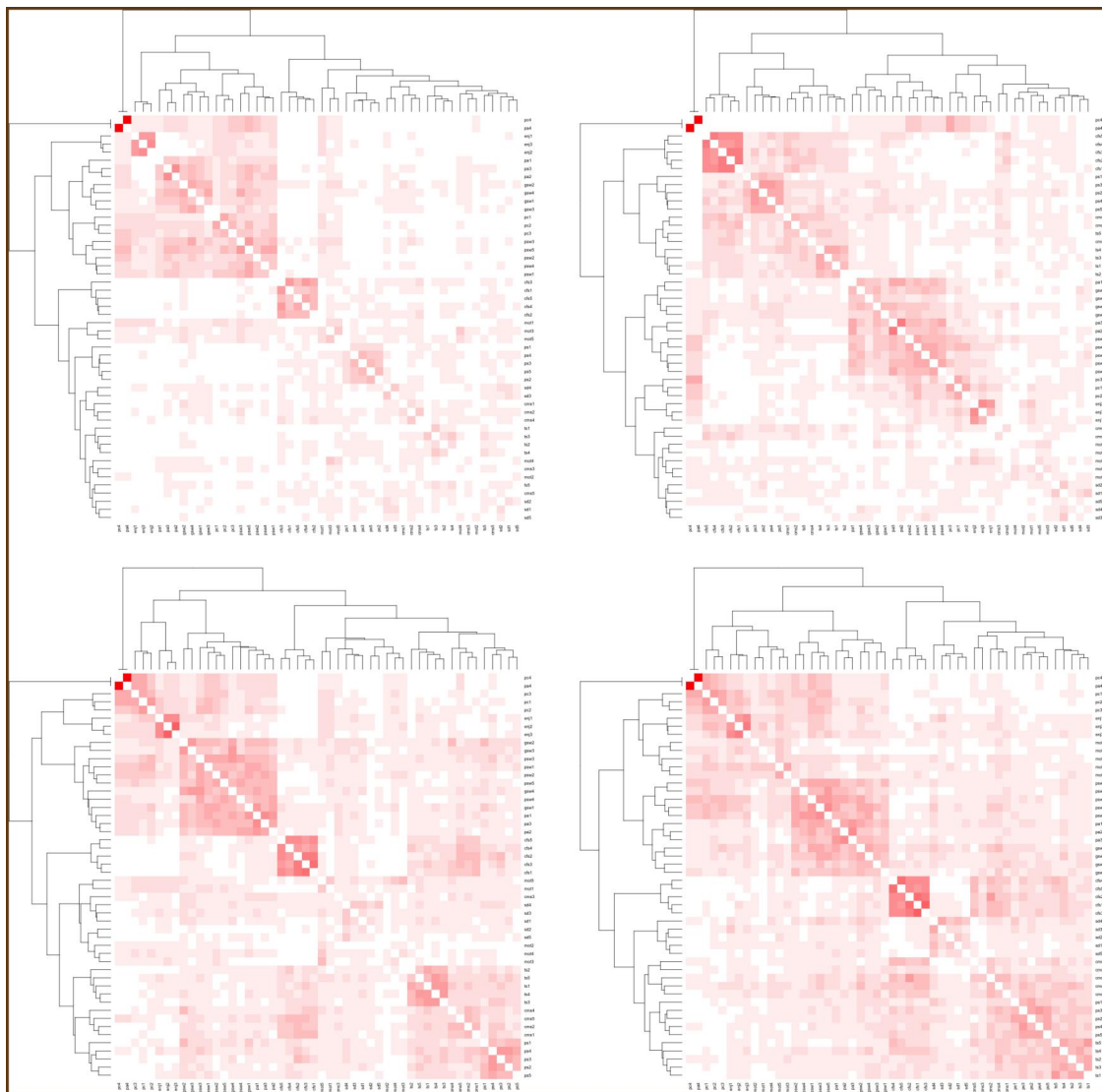
**Table A1.** Sample items for each of the scales.

| Scale                | Sample item                                                            |
|----------------------|------------------------------------------------------------------------|
| Perceived competence | Some teenagers feel that they are very athletic.                       |
| Perceived appearance | Some teenagers really like their looks.                                |
| Physical self-worth  | Some teenagers wish they could feel better about themselves.           |
| Global self-worth    | Some teenagers are happy being the way they are.                       |
| Parent support       | Some teenagers have parents who care about their feelings.             |
| Teacher support      | Some teenagers have a teacher who cares about them.                    |
| Close-friend support | Some teenagers don't have a close friend who listens to what they say. |
| Classmate support    | Some teenagers spend recess time with their class-mates.               |
| Motivation           | Some students are often discouraged after failures.                    |
| Enjoyment            | Some students enjoy being in their class.                              |
| Self-determination   | Some students feel they have a say in what they do.                    |

**Table A2.** Uncertainty of  $gsw1$  explained by domain-specific variable with total variations:  $1 - CE^{res}(gsw1|X_s)$  and  $\mathcal{V} < gsw1|X_s >$  for time-1.

|    | Covariate | $1 - CE^{res}[gsw1 X_s]$ | $\mathcal{V} < gsw1 X_s >$ |
|----|-----------|--------------------------|----------------------------|
| 1  | pa1       | 0.303                    | 0.048                      |
| 2  | psw1      | 0.249                    | 0.058                      |
| 3  | pc1       | 0.190                    | 0.056                      |
| 4  | cms1      | 0.143                    | 0.037                      |
| 5  | mot1      | 0.127                    | 0.040                      |
| 6  | enj1      | 0.065                    | 0.031                      |
| 7  | ts1       | 0.058                    | 0.005                      |
| 8  | sd1       | 0.054                    | 0.010                      |
| 9  | ps1       | 0.043                    | 0.007                      |
| 10 | cfs1      | 0.034                    | 0.006                      |



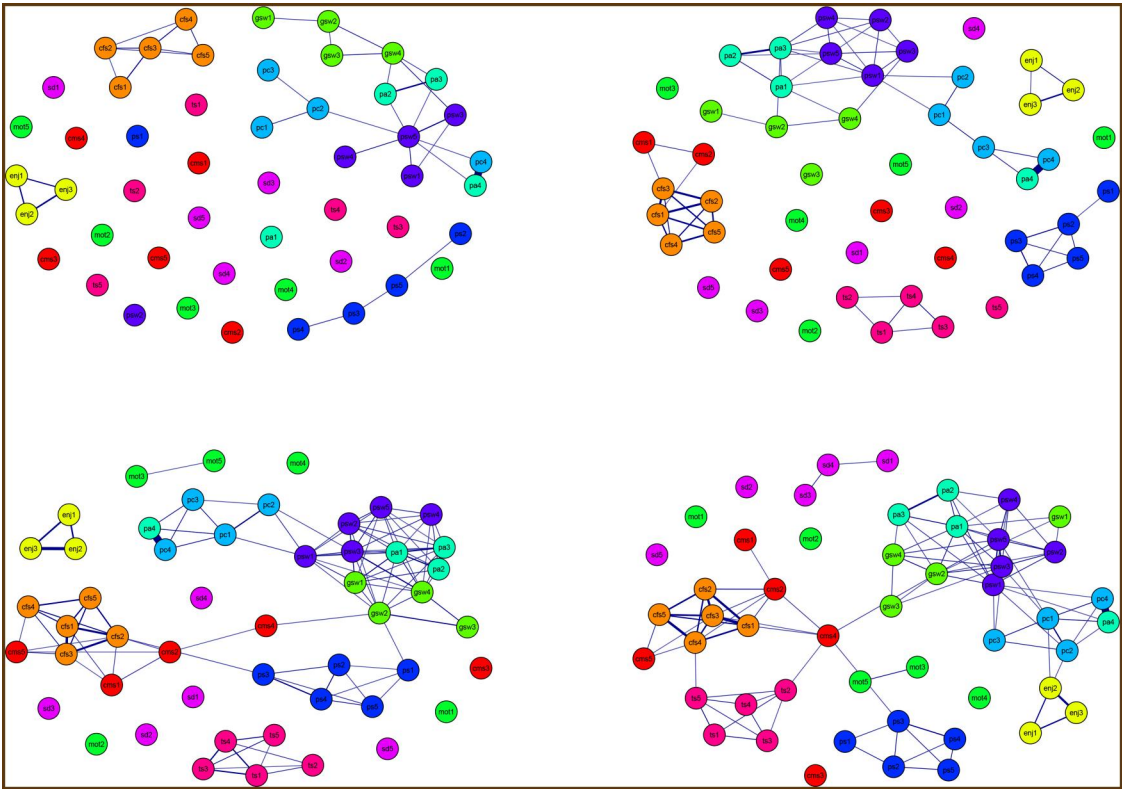


**Figure A1.** Four time-stamp specific heatmaps of MCEs among 50 items. From top left corner, clockwise: Time-1, Time-2, Time-3, Time-4.

**Table A3.** Number of subjects per response category (columns) across the four major covariates at time 1 (rows).

| Conditioning variable | Category = 1 | Category = 2 | Category = 3 | Category = 4 |
|-----------------------|--------------|--------------|--------------|--------------|
| pa1                   | 20           | 68           | 63           | 62           |
| psw1                  | 16           | 54           | 78           | 65           |
| pc1                   | 31           | 56           | 74           | 52           |
| cms1                  | 26           | 54           | 77           | 56           |

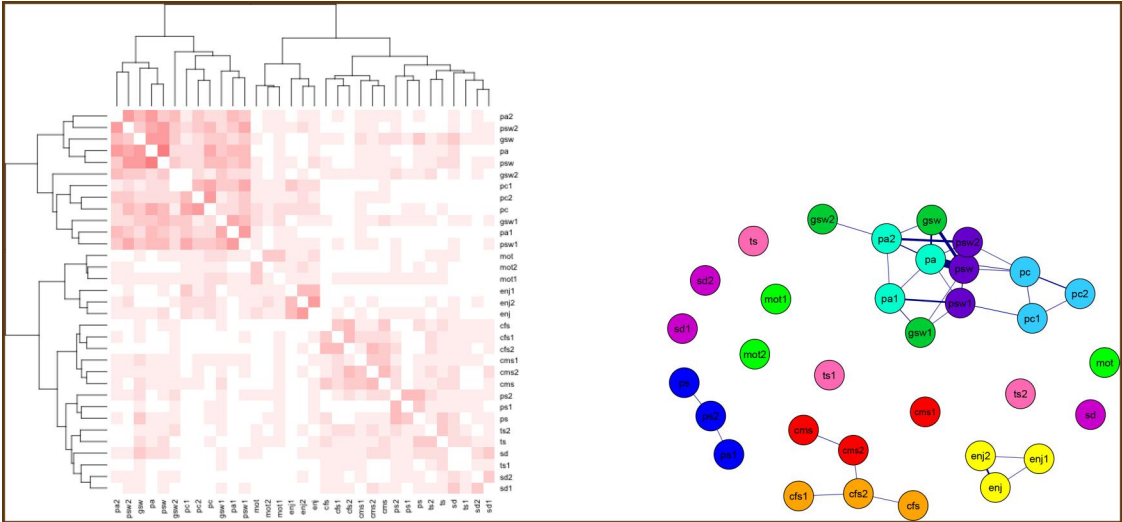




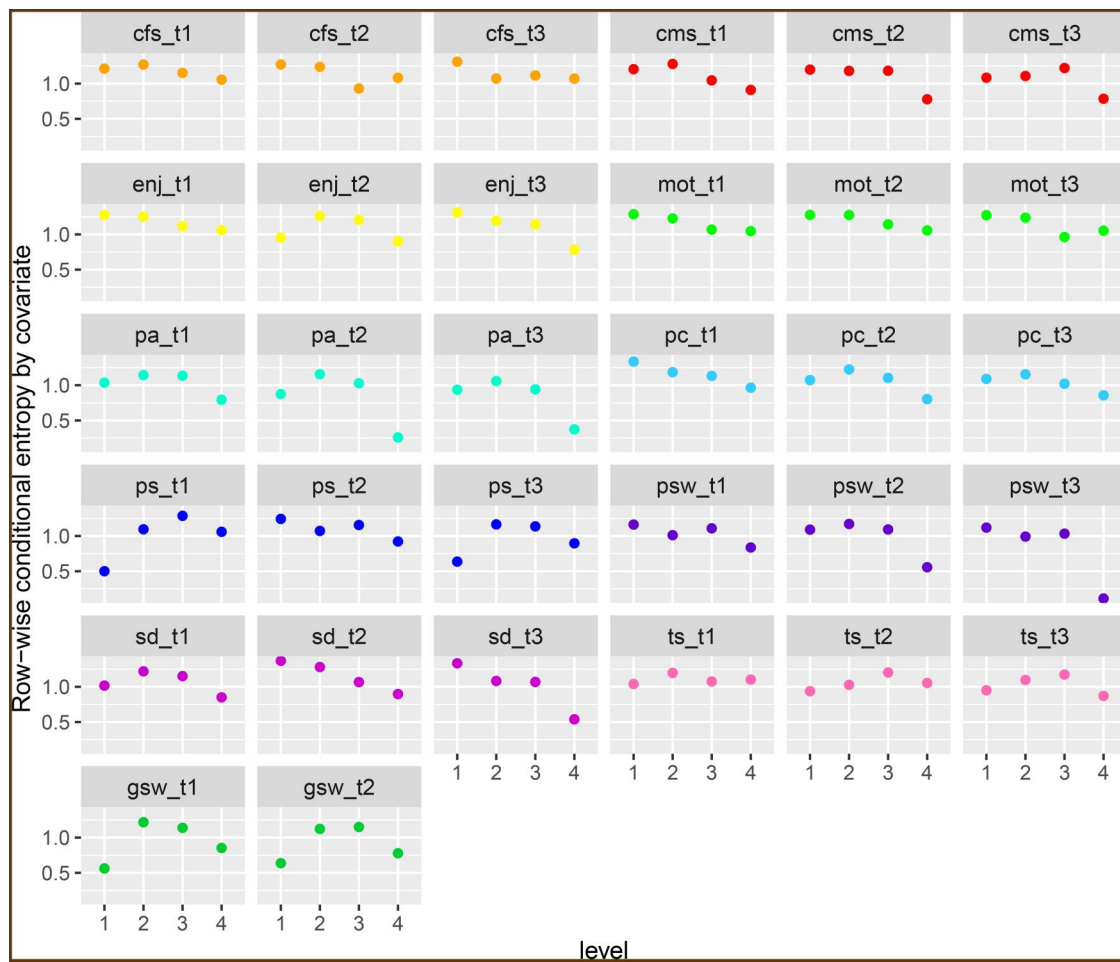
**Figure A2.** Four time specific networks of MCEs among 50 items. MCE represented as a network. From top left corner, clockwise: Time-1, Time-2, Time-3, Time-4. A linkage between nodes is presented if  $1 - MCE > 0.2$ .

**Table A4.** Triplets providing the most information (highest  $CE^{res}$  for each conditioning variable across each category at time 1.

| Conditioning variable | Category          |                   |                   |
|-----------------------|-------------------|-------------------|-------------------|
|                       | 1 OR 2            | 3                 | 4                 |
| pa1                   | (psw1, mot1, pc1) | (cfs1, cms1, sd1) | (cms1, cfs1, ps1) |
| psw1                  | (pa1, pc1, mot1)  | (cms1, pa1, sd1)  | (pa1, cms1, ps1)  |
| pc1                   | (pa1, psw1, mot1) | (cms1, pa1, sd1)  | (psw1, ts1, ps1)  |
| cms1                  | (pc1, psw1, pa1)  | (pa1, psw1, pc1)  | (psw1, pa1, mot1) |



**Figure A3.** Heatmap of  $33 \times 33$  matrix of  $1 - CE^{res}[gsw3|X_s]$  pertaining to 33 domain-specific variables at time-3 (Left panel). MCE represented as a network, a linkage between nodes is presented if  $1 - MCE > 0.2$ . (Right panel).



**Figure A4.** Row-wise conditional entropies of gsw3 given levels of each covariate  $H(gsw3|X_s = x_s)$ . The suffix “t” is added here to highlight the time point the variable refers to.

**Table A5.** Uncertainty of gsw3 explained by domain-specific variable with total variations:  $1 - CE^{res}(gsw3|X_s)$  and  $\mathcal{V} < gsw3|X_s >$  for time-3.

|    | Covariate | $1 - CE^{res}[gsw3 X_s]$ | $\mathcal{V} < gsw3 X_s >$ |
|----|-----------|--------------------------|----------------------------|
| 1  | psw3      | 0.411                    | 0.167                      |
| 2  | pa3       | 0.377                    | 0.078                      |
| 3  | pa2       | 0.312                    | 0.120                      |
| 4  | sd3       | 0.254                    | 0.071                      |
| 5  | psw2      | 0.209                    | 0.057                      |
| 6  | gsw2      | 0.200                    | 0.033                      |
| 7  | gsw1      | 0.193                    | 0.028                      |
| 8  | pc3       | 0.191                    | 0.012                      |
| 9  | psw1      | 0.191                    | 0.015                      |
| 10 | ps3       | 0.167                    | 0.018                      |
| 11 | pa1       | 0.165                    | 0.024                      |
| 12 | ts3       | 0.154                    | 0.019                      |
| 13 | ps2       | 0.152                    | 0.014                      |
| 14 | cms3      | 0.146                    | 0.028                      |
| 15 | pc2       | 0.142                    | 0.022                      |
| 16 | cms1      | 0.127                    | 0.019                      |
| 17 | enj3      | 0.110                    | 0.031                      |
| 18 | cms2      | 0.106                    | 0.029                      |
| 19 | sd1       | 0.104                    | 0.022                      |
| 20 | mot3      | 0.102                    | 0.015                      |
| 21 | cfs3      | 0.101                    | 0.003                      |
| 22 | sd2       | 0.090                    | 0.027                      |
| 23 | ts2       | 0.086                    | 0.008                      |
| 24 | ts1       | 0.086                    | 0.002                      |
| 25 | ps1       | 0.086                    | 0.022                      |
| 26 | mot1      | 0.078                    | 0.008                      |
| 27 | pc1       | 0.072                    | 0.013                      |
| 28 | cfs1      | 0.067                    | 0.006                      |
| 29 | enj2      | 0.062                    | 0.021                      |
| 30 | cfs2      | 0.060                    | 0.013                      |
| 31 | enj1      | 0.045                    | 0.007                      |
| 32 | mot2      | 0.044                    | 0.007                      |

**Table A6.** Number of subjects per response category (columns) across the four major covariates at time 3 (rows).

| Conditioning variable | Category = 1 | Category = 2 | Category = 3 | Category = 4 |
|-----------------------|--------------|--------------|--------------|--------------|
| psw                   | 13           | 27           | 75           | 42           |
| pa                    | 11           | 47           | 56           | 43           |
| patime2               | 17           | 56           | 49           | 35           |
| sd                    | 14           | 38           | 60           | 45           |

**Table A7.** Triplets providing the most information (highest  $CE^{res}$  for each conditioning variable across each category at time 3.

| Conditioning variable | Category          |                  |                   |
|-----------------------|-------------------|------------------|-------------------|
|                       | 1 OR 2            | 3                | 4                 |
| psw3                  | (gsw2, psw1, pc1) | (sd3, pa2, ts3)  | (cms2, cfs3, ps2) |
| pa3                   | (pc3, enj3, gsw2) | (sd3, ps2, cfs3) | (sd3, ps2, ts3)   |
| pa2                   | (ps3, psw3, ts3)  | (pc2, psw3, ps3) | (ps2, ts2, cfs3)  |
| sd3                   | (psw3, gsw1, pa3) | (pa2, psw3, pa3) | (pa3, psw3, pc3)  |